A LABORATORY INVESTIGATION OF THE APPLICATION OF TRANSFER FUNCTIONS TO FLEXIBLE PAVEMENTS

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BY

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Final Report

A LABORATORY INVESTIGATION OF THE APPLICATION OF TRANSFER FUNCTIONS TO FLEXIBLE PAVEMENTS

TO: J. F. McLaughlin, Director
Joint Highway Research Project

September 7, 1972

Project: C-36-6Z

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

File: 2-4-26

The attached Final Report titled "A Laboratory Investigation of the Application of Transfer Functions to Flexible Pavements" is submitted as fulfillment of the objectives of the Plan of Study approved by the Board on September 16, 1970, under title of "Response of Flexible Pavements to Static and Dynamic Loads Using Impulse Loading".

The research was conducted and the Report authored by Mr. G. A. Ali, Graduate Instructor in Research on our staff. Professors W. H. Goetz and M. E. Harr served as advisors and directors of this research effort.

The Final Report is submitted to the Board for acceptance as fulfillment of the objectives of the Study.

Respectfully submitted,

Harold L. Michael

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Project No.: C-36-6Z

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LIST OF SYMBOLS

A	Constant
C	Constant
C_i	Coefficients of the Dirichlet series
c	Viscosity coefficient
D	Constant
E_1	Elastic modulus of surface course
E_2	Elastic modulus of base course
E_3	Elastic modulus of subgrade
E_i	Exponents of the Dirichlet series
F_o	Magnitude of input forcing function
F_p	Peak value of impulse load
$F(t)$	Input forcing function
$\overline{F}(s)$	Laplace transform of $F(t)$
$G(s)$	Transfer function
$G(t)$	Response function
$G(t_k)$	Vector quantity of the response function at time t_k
h	Compacted thickness of layered material
$I(t)$	Input
$I(t_k)$	Value of input at time t_k
$\overline{I}(s)$	Laplace transform of $I(t)$
K	Conversion factor

k	Spring constant
L	Laplace transformation
L^{-1}	Inverse Laplace transformation
m	Mass
mc	Sensitivity of Brush recorder
N	Number of repeated load applications
n	Number of terms in the Dirichlet series
$O(t)$	Output
$O(t_k)$	Value of output at time t_k
$\overline{O}(s)$	Laplace transform of $O(t)$
p	Period of haversine loading function
p_i	Poles of the transfer function
R	Programmed load range of MTS machine
r_i	Zeros of the transfer function
s	Laplace transform parameter (= complex variable)
t	Time
t_n	Time at which impulse load first becomes non-zero
Δt	Time interval
W	Weight of layered material
x	Distance from load center
y_0	Central deflection of the deflected basin
y_p	Repeated load permanent response
$y(x)$	Deflection at distance x from load center
y_T	Repeated load total response
$y(t)$	Output displacement; static load response
$\dot{y}(t)$	Velocity

$\ddot{y}(t)$	Acceleration
$\overline{y}(s)$	Laplace transform of y(t)
$\alpha,\beta,\gamma,\delta,\varepsilon$	Constants
γ_{wet}	Density of compacted layered material
ϕ	$= \tan^{-1}\left(\frac{\gamma}{\beta}\right)$
ϕ_1	$= \tan^{-1}\left(\frac{\gamma-\omega}{\beta}\right)$
ϕ_2	$= \tan^{-1}\left(\frac{\gamma+\omega}{\beta}\right)$
ω	Frequency

ABSTRACT

Ali, Galal Abdalla. Ph.D., Purdue University, August 1972. A Laboratory Investigation of the Application of Transfer Functions to Flexible Pavements. Co-Major Professors: William H. Goetz and Milton E. Harr.

Transfer function theory was applied to examine the behavior of flexible pavements. Two convolution techniques, one numerical and the other analytical, were employed. The numerical convolution was used to derive pavement response functions from impulse testing of three-layer, flexible model pavements. The analytical convolution formed the basis for calculating deflections resulting from static and repeated loads.

It was hypothesized that more significant parameters than commonly used could be obtained under controlled laboratory conditions. By using non-linear regression, the response functions were approximated by a mathematical model to include these parameters. The derived model was used as input in the analytical convolutions. The adequacy of the developed models was verified by comparing predicted and measured deflections.

Curve-fitting of the dynamic peak deflection data, by the non-linear regression, resulted in a mathematical function for the deflection basin.

A silty sand subgrade, a crushed aggregate base, and an asphalt concrete surface constituted the components of the three-layer systems. Each layer material was characterized by a series of conventional laboratory tests. Model pavements, of two different surface course thicknesses, were tested statically and dynamically at three different stress levels. Test temperatures were 50, 75 and 100°F. Deflections were measured at five spatial locations.

The following are the primary findings of this research:

1. The profile of peak deflections of a flexible highway pavement can be described by the equation

$$y(x) = y_0 e^{-Dx^2}$$

where $y(x)$ is the deflection at a distance x from the load center, y_0 is the central deflection and D is a constant reflecting the attenuation of the deflection profile with x.

2. The central deflection y_0 increases with increases in temperature, and decreases with increasing surface course thickness. Increases in temperature (50°F - 100°F) or surface course thickness (1 inch and 2 inches) decrease the value of the parameter D.

3. The time-dependent behavior of a flexible pavement can be represented by a response function $G(t)$ which is a function of time t. It is possible to obtain this function from a single impulse test on the pavement. The response function is independent of the magnitude of the impulse load.

4. The response function of a flexible highway pavement is of the form

$$G(t) = \alpha e^{-\beta t} \sin \gamma t$$

where α, β and γ are descriptive parameters in the response function.

5. Parameters in the response function are believed to be pavement properties that provide for better understanding of its behavior than those currently used.

6. Temperature, surface course thickness and spatial location have their respective influences on the response function. Increases in temperature increase the value of the α parameter in the response function, while increasing surface course thickness or the distance from the load center decreases the value of α. The β and γ parameters do not seem to be affected appreciably by the above factors.

7. Transfer function theory is capable of predicting static or repeated load deflections of flexible pavements. The favorable agreement between predicted and measured values of the deflections in this study validates the hypothesis that the parameters in the response function are material descriptors which are independent of the type of load input.

INTRODUCTION

The pursuit of a rational method for designing and evaluating pavements requires considerable knowledge of their response behavior to realistic loadings. The ability of a pavement structure to withstand such imposed loads can be expressed in terms of primary and ultimate response modes [1, 2].* In the primary response, the magnitudes of developed stresses, strains or deflections can be used, whereas failure must be considered in the ultimate response. Hence, an essential part in pavement design is the development of a procedure to analyze a pavement system for representative input stimuli. Another necessary requirement is to define a failure criterion in terms of the ultimate responses, for example by limiting deflection, so that the designer may assign allowable values to these quantities. When deflection is considered as the criterion, it must be supplemented by other measurements, such as the slope of the deflection profile. Knowledge of the analytic form of the deflection profile is a prerequisite to determining its slope.

Although the question as to what causes failure in asphaltic concrete pavements is a complex one, several failure mechanisms have been postulated [3, 4, 5, 6].

---

* Numbers in brackets refer to entries in the Bibliography.

Surface cracking and rutting may be caused by the effect of direct wheel loads, shear displacements and further compaction of the base structure, or due to irrecoverable deformations in the subgrade material resulting in creep or plastic flow. The elastic or recoverable portion of the deformations in the bottom layers under the action of repeated loads may result in fatigue failure, whereas insufficient subgrade shear strength can lead to shear type failure. Brittle surface cracking may be caused by non-load-associated stresses such as those effected by changes in temperature.

Current mathematical models to predict pavement response and to evaluate its behavior generally evolve from elastic and viscoelastic theories. These models incorporate geometric aspects and material properties within a system of equations which, in turn, are solved so as to satisfy selected boundary conditions. However, any model, required to be predictive, must be inclusive of realistic in-situ material properties. Also, pavement materials, and particularly bituminous concrete, are time- and temperature-dependent. At the present time no single theory of pavement behavior that considers time- and temperature-dependent characteristics of pavement systems is used in designing or evaluating pavements. Thus, a need exists to obtain parameters or functions capable of completely describing the time- and temperature-dependent material properties of pavements subjected to static or dynamic loads.

A new approach undertaken in the present study to evaluate pavement behavior deals with the primary response mode. The basis of the investigation is the transfer function theory that has been applied successfully in the synthesis and design of various systems in other fields of engineering.

This investigation extends the transfer function concepts to flexible pavements. More significant material properties, or indicators, than commonly hypothesized are defined and determined under controlled laboratory conditions. The adequacy of the developed mathematical-experimental models is evaluated by comparing predicted responses due to static and dynamic loads with experimentally measured values. The effect of surface course thickness, temperature and spatial location on the dynamic parameters and the predictions are examined.

In addition, it is hypothesized that the deflection basin of a pavement follows a normal distribution curve. A mathematical form is obtained to fit the deflection basin.

The procedure to achieve the above objectives as a pavement evaluation tool is depicted in the flow chart of Figure 1. A known impulse load is applied to a pavement system. The resulting output deflections are measured as functions of time at several locations on the pavement surface. The peak deflections are curve-fitted using a computer program NONLINR. A mathematical function is thus

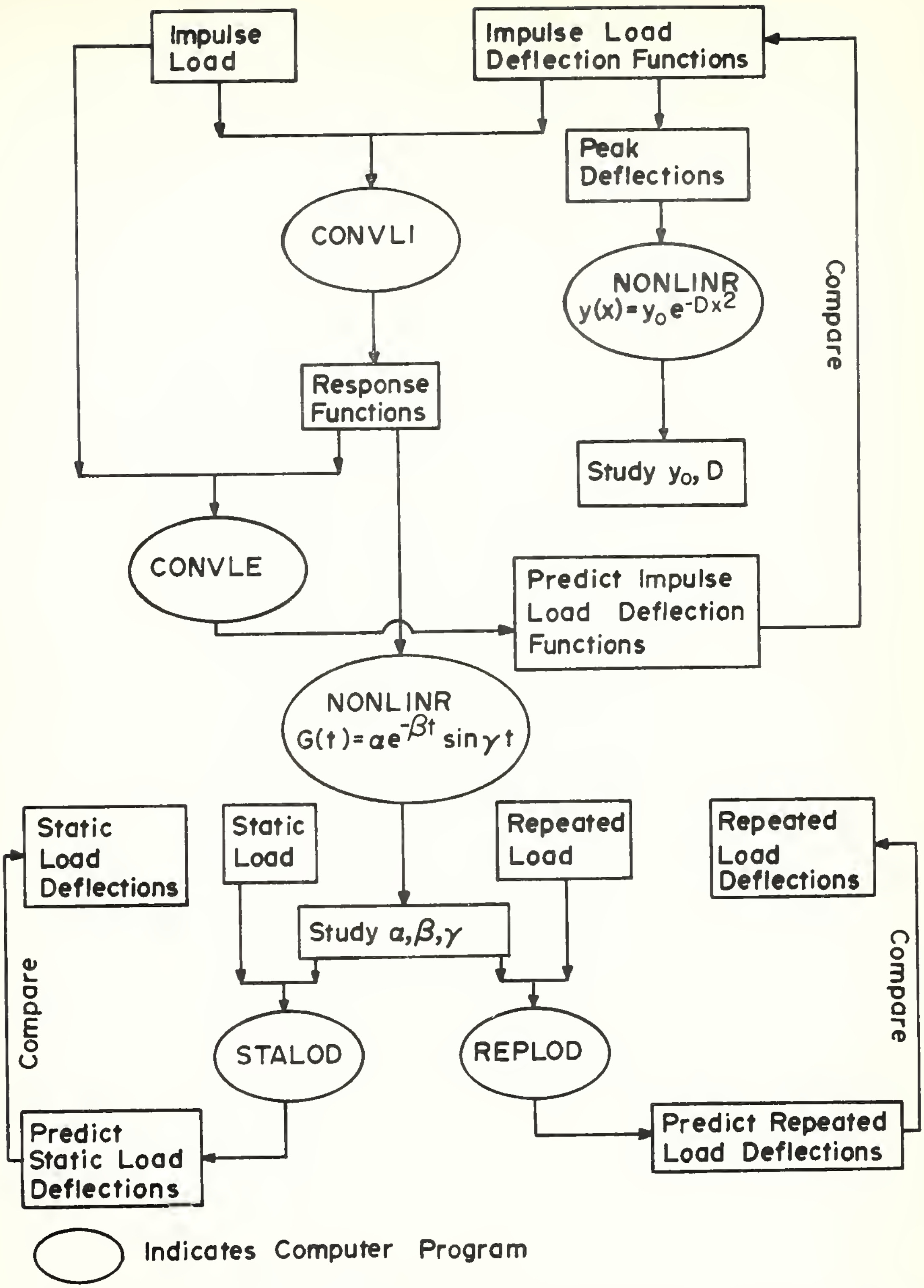


FIGURE 1 PROCEDURE FOR PAVEMENT EVALUATION

obtained to describe the deflection basin. The parameters in this function are studied.

The time-dependent input and outputs resulting from the impulse tests are analyzed within a formulated theory to give response functions. A computer subroutine CONVLI is employed. The response functions are verified using subroutine CONVLE.

A mathematical model for the response functions is evolved through NONLINR. This model is convoluted analytically with step and repeated load functions. Computer programs STALOD and REPLOD facilitate the respective calculations. Predicted and measured responses are compared.

obtained to describe the deflection basin. The parameters in this function are studied.

BACKGROUND AND STATE OF THE ART

In this section a literature review is presented which develops the necessary background and state of the art for this investigation. Five aspects of material characterization and pavement studies pertaining to this study are presented:

1. Application of elastic theory.
2. Developments from viscoelastic theory.
3. Use of dynamic models.
4. Transfer function approach.
5. Laboratory studies of flexible pavements.

Application of Elastic Theory

One of the earliest applications of the theory of elasticity to an infinite layer medium was the investigation reported by Boussinesq [7]. Later, Burmister [8, 9] presented the stresses and displacements in two- and three-layered systems. The layer theory was further extended by Fox [10], Acum and Fox [11], Mehta and Veletsos [12], Jones [13] and others. Generally, all these theories have in common the following assumptions: (a) Each of the system layers is a continuous, homogeneous, isotropic, linearly elastic medium of infinite horizontal extent; (b) each layer

is continuously in contact with, and supported by, the underlying layer; (c) the surface loading can be represented by a vertical pressure uniformly distributed over a circular area; (d) inertia forces are negligible; and (e) displacements are small. That no pavement layer can satisfy these restrictions, except for extreme cases, is generally accepted. Furthermore the above theories do not consider dynamic loading, since the stress function technique, which underlies these methods, is applicable only to static loads. Avramesco [14] outlined a mathematical solution for computing displacements in elastic layered structures. The method accounts for the dynamic phenomena in pavements and, according to Avramesco, can be extended to the viscoelastic case. However, only qualitative description of the dynamic behavior was reported without numerical results.

How closely the material properties are represented in any theory is reflected in its predictive capability. The representation of material properties by two elastic constants (as is required by elastic theory) is simply a mathematical expedient. From laboratory tests on specimens of bituminous mixtures, Busching, Goetz and Harr [15] determined six independent coefficients necessary to describe the stress-strain relationships considering the material as homogeneous and transversely anisotropic when subjected to small strains.

Despite the several shortcomings that elastic layer theory has, this model can be used as a first approximation.

However, it remains to be determined how the performance of pavements in the real world is related to the prediction of stresses in idealized elastic pavement systems.

Developments from Viscoelastic Theory

For several years the component materials of a pavement system have been recognized to have time dependent stress-strain properties due to consolidation and creep [16, 17]. Thus, to overcome the limitations imposed by the assumptions of time-independent elastic materials, the viscoelastic approach was introduced into material characterization and pavement analysis.

Viscoelastic model representations of soils have been discussed by Schiffman [18]. Lee [19] and Lee and Rogers [20] presented more general methods of viscoelastic stress analysis. Secor and Monismith [21] studied the viscoelastic properties of bituminous concrete using a four element model. Papazian [22] represented asphaltic concrete by a mechanical model consisting of five Kelvin units in series with one Maxwell unit. He pointed out that an infinite number of Kelvin units would be needed for accurate representation of the material.

The development of the correspondence principle for isotropic media in 1955 by Lee [23], and shortly thereafter its extension by Biot [24] to include anisotropic media, made possible the analysis of pavements using time dependent material properties. The viscoelastic solutions are based

on the premise that the time dependent boundary value problem can be reduced to an elastic model if all time varying quantities are replaced by their equivalent Laplace transforms and the elastic constants by operator forms of the stress-strain relations. Perloff and Moavenzadeh [25] determined vertical deflections for a static load and a point load moving at a constant velocity on the surface of a homogeneous, isotropic, linear viscoelastic half-space.

The solutions to two-layer systems, where either one or both components were considered viscoelastic, was the subject of the papers published by Pister and his co-workers [26, 27, 28, 29]. The problem of a viscoelastic asphaltic concrete beam on a set of independent springs approximating a Winkler-type foundation was investigated for both a stationary load [26] and a concentrated moving force [27]. The case when the top layer was assumed to be elastic and the subgrade to be viscoelastic was solved by Pister and Williams [28] and Hoskin and Lee [30]. Both layers were considered to have time dependent (viscoelastic) properties in the analysis presented by Pister [29] and Huang [31].

In response to the demand of research workers for improvements on pavement models to include realistic material representation, more elaborate models were used to solve the three- and four-layer viscoelastic problems [2, 31, 32]. Barksdale and Leonards [33] presented a viscoelastic analysis for three- and four-layer pavement systems subjected to

repeated loads. An attempt was made to compare the theoretical predictions of the resilient and permanent deformations due to repeated loads with those determined at the AASHO Road Test. The measured permanent surface deflection for the AASHO Road Test Section was more than three times the predicted value. The discrepancy was mainly due to the assumption of an elastic base course made in the viscoelastic analysis, while the results of the AASHO Road Test indicated that 59 percent of the permanent deformation occurred in the base and subbase. Thus, in predicting flexible pavement performance, all layered materials must be considered to have time dependent characteristics.

Based on the fact that pavement materials are viscoelastic and temperature-dependent, Ku [34] proposed stress-strain relations that accounted for temperature variations. Although the temperature-dependent relations were verified experimentally [35, 36], the application of the stress-strain-temperature constitutive relations to the solution of layered systems has yet to be developed. Recently, Moavenzadeh [37] extended the primary response model for a three-layer viscoelastic system [2] to include failure in flexible pavements. The developed mathematical formulation accounts for environmental conditions, such as temperature and moisture, but no numerical results were provided.

When different temperatures are being considered, the equivalence of spring and dashpot models are lost. To obtain

the temperature-dependent constitutive relations for visco-elastic materials, either more simplifying approximations must be made or data used directly [38].

Each of the above investigations, within the framework of viscoelastic theory, attempted to take into consideration the time dependence of pavement materials and consequently provided an insight into their behavior; however, the correspondence principle requires that the associated elastic problem must be amendable to solution. The elastic approach, in turn, imposed certain mechanistic material properties (constitutive relations) into the analysis inasmuch as the equations obtained from equilibrium conditions, strain definitions, and compatibility relations are less than the number of unknowns contained within these relationships.

In both elastic and viscoelastic theories, mathematical expediency has led to idealization of the constitutive relations. The combination of springs and dashpots selected to characterize the stress-strain properties of pavement layer materials depends on the individual investigator(s). Basically, the criterion is to obtain a better curve fit for the response sought. Usually, some difficulty is involved in taking the inverse Laplace transform to obtain the final solution.

Among the viscoelastic models commonly used are the Maxwell [39], Voigt-Kelvin [25], van der Poel (or Standard Solid) [40], four-element [21, 41] or combinations of these

models [22, 31, 33]. The model constants are usually evaluated by running a creep test. The response to a unit step load is then approximated by the Dirichlet series

$$\sum_{i=1}^{n} C_i e^{-tE_i}$$

where C_i and E_i are constants, t is time and n is the number of terms desired. The exponents E_i are given specific values as suggested by Shapery [42] and the coefficients C_i are determined by the linear least squares method of curve fit. It is noticed here that the material parameters C_i are predetermined by the experimenter so as to avoid the non-linear set of equations that result if the error sum of squares is minimized with respect to each parameter [42].

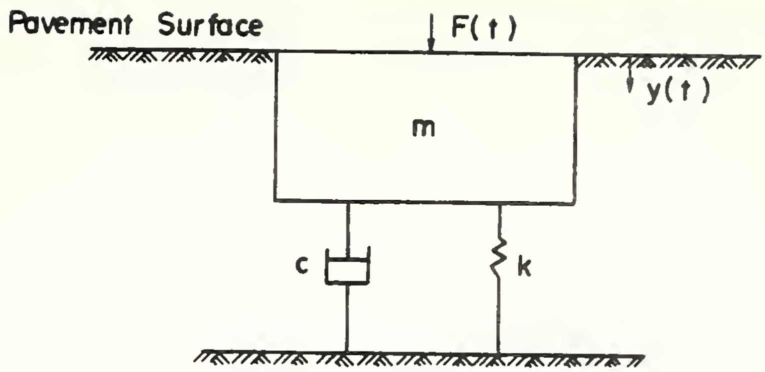
One of the objectives of the present investigation is to determine the material properties defined by the response to a unit impulse load using a non-linear least squares curve fitting technique. A computer subroutine [43] is employed for this purpose.

Use of Dynamic Models

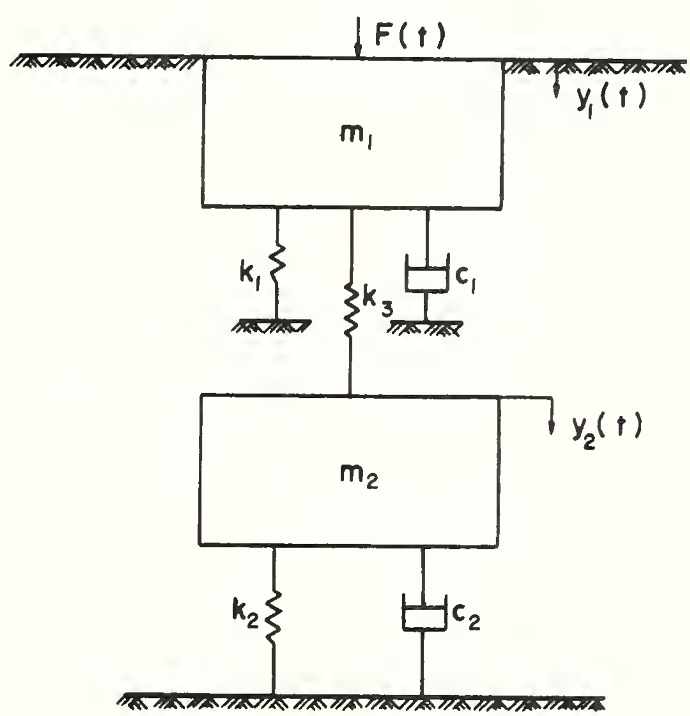
Recently, research workers have recognized the fact that pavements should be regarded as dynamic systems. The elastic and viscoelastic theories do predict, within a limited degree of accuracy, static and quasi-static stresses and displacements, but little is known about the dynamic behavior of pavements. Lattés, Lions and Bonitzer [44] attempted to

include inertia effects but the method was in the development stage and no numerical results were published.

In dynamic models, the mass, dashpot and spring function represent, respectively, inertia, damping effect and restoring force, of a mechanical system. The two configurations mostly used to represent a road structure are shown in Figures 2-a and 2-b, respectively, for a single degree of freedom (three-parameter) model and a seven-parameter model. Using sustained vibrations and assuming the mechanical system of Figure 2-a to represent a road system, Heukelom [45] analyzed dynamic deflections of pavements. Harr [46] used a three-parameter model to study the effect of vehicle speed on dynamic pavement deflections due to a rectangular pulse loading function. In conjunction with their study to calculate the transient responses of a pavement system, Szendrei and Freeme [47] experimentally determined the shape of the impulse to which a road is subjected under traffic loading. The two pulse shapes which were observed are illustrated in Figure 3. Simple as well as complicated models [47, 48, 49, 50] have been proposed to investigate the dynamic response of pavement systems. The inadequacy and shortcomings of such models have been discussed in the literature [45, 48, 49, 50]. The fact that these models do not possess sufficient predictive capability to satisfy either existing or contemplated needs is not surprising since the representation of pavement behavior by a spring-mass-dashpot system is only a simple

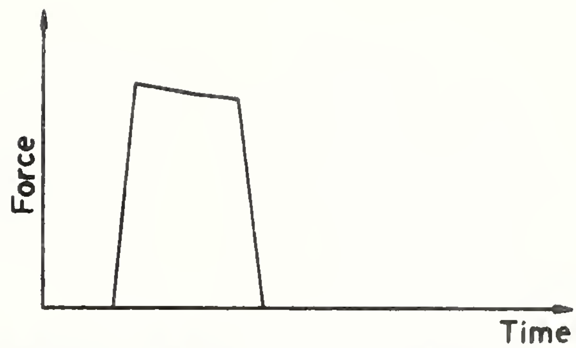


a. Three - Parameter Model

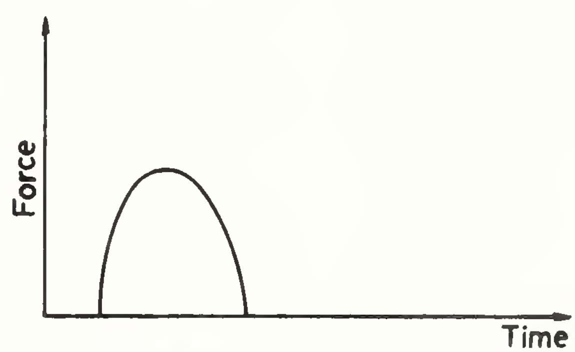


b. Seven - Parameter Model

FIGURE 2 MECHANICAL MODELS COMMONLY USED



a. Rectangular Pulse Near the Tire Center



b. Half - Sine Pulse Near the Tire Edge

FIGURE 3 TYPICAL PULSE SHAPES PRODUCED BY A MOVING WHEEL LOAD (47).

idealized model in which distributed effects are approximated by lumped parameters. This is attributed to lack of a true model that would satisfactorily serve the purpose.

Transfer Function Approach

Based on transfer functions, several solutions to mechanical and electrical engineering problems are found in the literature [51, 52, 53, 54]. The transfer function represents the operator which, acting on the time-dependent input, yields the output of a dynamic system. When obtaining the transfer function analytically, the system parameters are defined by assumed differential equations. For example, the equation of motion for the system shown in Figure 2-a is given by

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \tag{1}$$

where m = mass,

c = viscosity coefficient,

k = spring constant,

$y(t)$ = output displacement

$\dot{y}$ = velocity,

$\ddot{y}(t)$ = acceleration,

$F(t)$ = input forcing function, and

t = time.

The transfer function $G(s)$ for this three-parameter system is obtained from the ratio of the Laplace transform of the output $\overline{y}(s)$ to that of the input $\overline{F}(s)$ as

$$G(s) = \frac{\bar{y}(s)}{\bar{F}(s)} \tag{2}$$

where s is a complex variable.

Neglecting the initial conditions, G(s) becomes

$$G(s) = \frac{1}{ms^2 + cs + k}$$

$$= \frac{A}{(s-p_1)(s-p_2)} \tag{3}$$

where $A = \frac{1}{m}$, and (4a)

$$p_1, p_2 = -\frac{c}{2m} \pm \sqrt{\frac{c^2}{2m} - \frac{k}{m}} \tag{4b}$$

Similarly, the differential equations of motion for the seven-parameter model (Figure 2-b) are written as [55]

$$m_1\ddot{y}_1(t) + c_1\dot{y}_1(t) + (k_1+k_3)y_1(t) - k_3y_2(t) = F(t) \tag{5a}$$

$$m_2\ddot{y}_2(t) + c_2\dot{y}_2(t) + (k_2+k_3)y_2(t) - k_3y_1(t) = 0 \tag{5b}$$

and the transfer function becomes

$$G(s) = \frac{\bar{y}_1(s)}{\bar{F}(s)}$$

$$= \frac{A(s-r_1)(s-r_2)}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)} \tag{6}$$

where the constant A, the zeros r_i and the poles p_i are functions of m_1, m_2, c_1, c_2, k_1, k_2 and k_3.

In general, the mathematical model from any mass-spring-dashpot mechanical system can be written in the form [52]

$$G(s) = \frac{(s-r_1)(s-r_2) \cdots (s-r_m)}{(s-p_1)(s-p_2) \cdots (s-p_n)} \tag{7}$$

However, the assumed mechanical model specifies the number of material parameters obtained in the mathematical representation.

The procedure used by Swami [56] to obtain the transfer function for asphaltic concrete presents a method where no mechanical model is assumed. The technique is based on direct frequency tests which require steady state conditions between the input and output when both vary sinusoidally with time. The amplitude and phase relationship between input and output sine waves are measured and the process is repeated throughout the frequency range of interest. The ratio of the magnitude of the output (displacement) to that of the corresponding input (force) is then plotted (in decibels) against the log of frequency to obtain the frequency spectrum. The asymptotic approximation of the frequency spectrum gave the following transfer function of the system:

$$G(s) = \frac{A(s-r_1)(s-r_2)(s-r_3)}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)} \tag{8}$$

The study [56, 57] concluded that an asphaltic concrete at a constant temperature could be uniquely represented by a transfer function; and that the coefficients in the transfer

function were believed to be better indicators of material performance than those previously used.

The frequency response method of determining the transfer function may be impractical in cases where short testing time is necessary. Some established pavement systems, such as Interstate Highways, may not tolerate off-schedule operation for steady-state sinusoidal measurements. In the present investigation the impulse response of a pavement system is obtained in the time domain from a single impulse test with a duration of less than one second.

Laboratory Studies of Flexible Pavements

Literature was reviewed to provide background in selecting the pertinent factors and assigning typical values to the variables used in this study. Results of the laboratory studies of others are pertinent in this regard.

Subbaraju [58] conducted model studies of stresses in the upper layers of pavements. A $2\frac{1}{2}$ - inch thick asphaltic concrete slab was placed on a subgrade soil confined in a wooden box with inside dimensions $23\frac{1}{2}$ by $25\frac{1}{2}$ by 23 inches deep. Static loads of 650 pounds were applied through a $3\frac{3}{4}$ - inch diameter bearing plate. Assuming the stresses for the loading used would not extend more than ten inches beyond the center loading point, no edge effect was considered. The assumption was verified by his test results.

Monismith and Secor [41] attempted to validate viscoelastic theory by analyzing a thin viscoelastic plate on a Winkler foundation. Theoretical deflections were compared with those measured from a 40 by 40 by $1\frac{1}{2}$ inches deep asphaltic concrete slab resting on a bed of closely spaced springs. In some instances, discrepancies, which were larger at higher test temperatures, were attributed to the assumptions made in the analysis.

To validate the discrete-element solutions for pavement slabs, Agarwol and Hudson [59] tested a simple two-layer model. The system used in the investigation consisted of a small aluminum slab seated on a saturated clay subgrade in a 2 by 2 by 2-foot wooden box. Load magnitudes of 100 and 200 pounds were transmitted statically through a $\frac{3}{4}$ - inch diameter rod.

Model studies where all layer materials were ideally simulating [60] or actually representative [61, 62] have been conducted. Surface course horizontal dimensions ranging from 9 by 9 inches [59] to 10 by 10 feet [62] have been used.

In comparison with static tests, dynamic tests have been relatively few in number. Brown and Pell [63] tested a two-layer system subjected to a pulse load having an amplitude of approximately 7.6 kips and durations of 0.1 and 2 seconds. The study confirmed that the assumptions of elastic theory are not generally valid for pavement materials

[63]. Waterhouse [61] tested a three-layer test pit to failure under a repeated load of 5 tons magnitude and frequency of 8 cycles per minute. The sides of the box containing the 6-foot square by 5-foot deep specimen were believed to have only a slight effect upon the results [61].

Drennon and Kenis [62] considered pavements as linear systems and applied Duhamel's superposition integral to predict repetitive load displacements from static loads. The maximum load intensity applied to the full-scale asphaltic concrete pavement model was 80 psi at a frequency of one cycle per second. It was concluded from the experimental results that the total observed deflections increased linearly with load and that the dynamic components of both deflection and strain were basically linear. Some discrepancy existed between measured and predicted displacements partly due to using only the creep curve rather than both creep and recovery curves as input to the superposition integral [62].

A promising field test to evaluate flexible pavements by impulsive loads was carried out by Isada [49, 50]. From the peak displacement and the magnitude of the impulse, a road stiffness was determined but no analysis of the displacement time data was performed.

The above review indicates that neither the classical theories (elastic or viscoelastic) nor the assumed dynamic models can adequately characterize the time-dependent response of a pavement system. It also shows that the

transfer function theory appears to be a sufficient framework within which the time-dependent behavior of pavements can be investigated.

transfer function theory appears to be a sufficient frame-work within which the time-dependent behavior of personality can be investigated.

THEORETICAL ANALYSIS

The basic concepts underlying the transfer function theory is presented in this section. Three areas are of major interest: general assumptions, experimental determination of the impulse response and prediction theory.

Assumptions

Transfer function theory is based on the assumption that pavements behave as deterministic systems. A relationship which exists between known input and output conditions is sought. This is in contradiction to classical theories which assume this relationship to be known initially. The system functions are schematically shown in Figure 4. Complex dynamic situations including aircraft flight data have been synthesized utilizing the assumption that a determinable function relates the output to the input [51, 52, 55].

Furthermore, it is assumed that pavements will behave as linear systems. Although at high magnitudes of stresses and temperatures non-linearity may be observed, the investigations reported by Heukelom [45], Drennon and Kenis [62] and others [47, 48, 61] have shown that the assumption holds for small deflections normally experienced by pavement systems. The AASHO Road Test results, where single as well

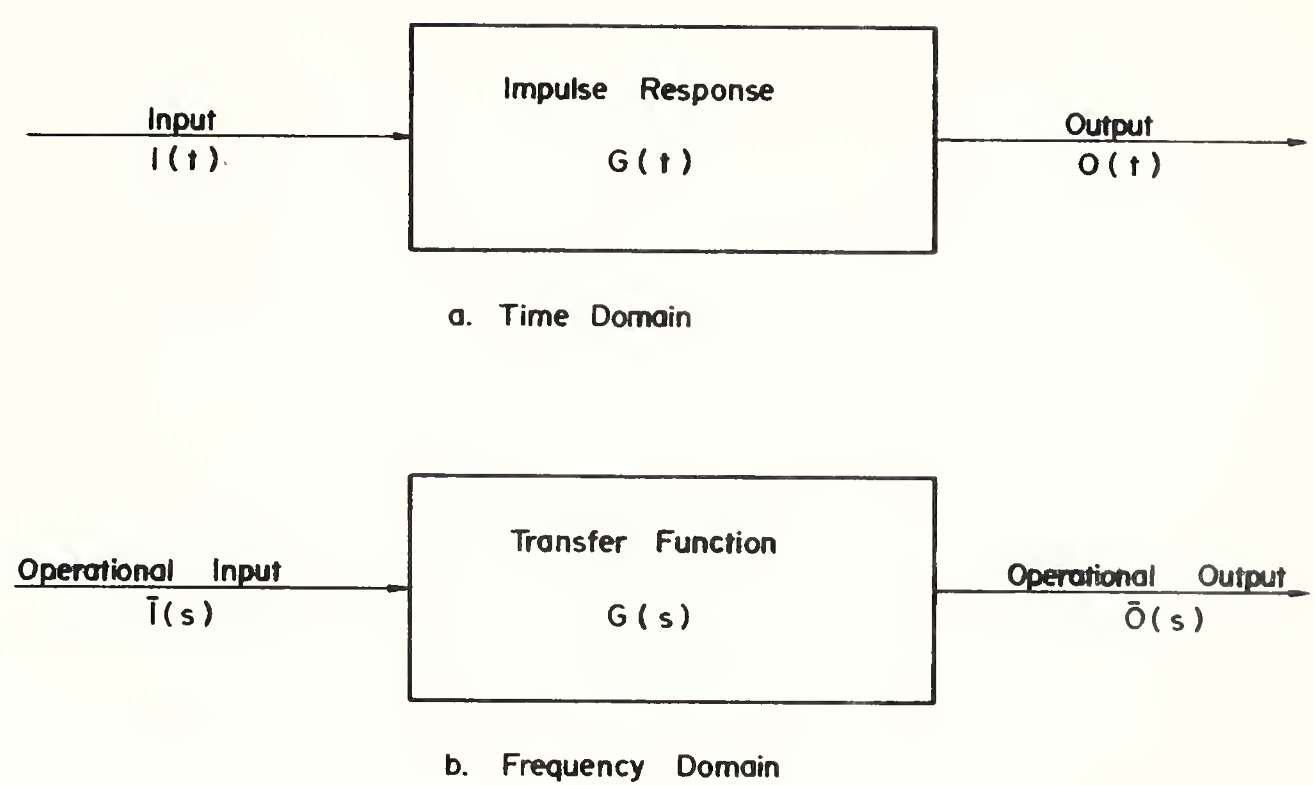


FIGURE 4 SCHEMATIC REPRESENTATION OF THE SYSTEM FUNCTIONS

as tandem axle loads were employed, indicated that creep speed deflections were linear in most cases [3, pp. 102-103].

Experimental Determination of the Impulse Response

With reference to Figure 4, a dynamic system can be represented by a transfer function G(s) defined as the ratio of the Laplace transform of the output O(t) to the Laplace transform of the input I(t). By this definition,

$$G(s) = \frac{\overline{O}(s)}{\overline{I}(s)}$$

or

$$\overline{O}(s) = G(s)\ \overline{I}(s) \tag{9}$$

where $\overline{O}(s) = L[O(t)]$ (10a)

$\overline{I}(s) = L[I(t)]$ (10b)

s = Laplace transform parameter,

t = time, and

L = Laplace transformation

The inverse transform of G(s) is

$$G(t) = L^{-1}[G(s)] \tag{11}$$

where L^{-1} is the inverse Laplace transformation.

The function G(t) is determined solely by the system parameters and is called the impulse response or the response function of the system. The operational form of Equation 9 can be expressed in the time domain using Borel's theorem on the convolution of two functions [54] to obtain the output as

$$O(t) = \int_0^t G(\tau)\ I(t-\tau)d\tau \qquad (12)$$

G(t) is obtained implicitly by the finite difference method [64] as

$$G(t_k) = \frac{O(t_k) - \sum_{j=1}^{k} G(t_j)\ I(t_{k-j+1})\ \Delta t}{KF_p\ \Delta t} \qquad (13)$$

where $G(t_k)$ = the vector quantity of the response function at time t_k,

$O(t_k)$ = the value of the output at t_k,

$I(t_k)$ = the value of the input at t_k,

K = conversion factor,

F_p = the peak value of the input, and

Δt = time interval.

The denominator in Equation 13 may be too small for Equation 13 to converge. If this is the case, as was found to be generally true in the present investigation, K is chosen such that the response function is within the conversion region. In this study, a denominator value of 9 or more would cause Equation 13 to converge. The K factor was taken to be 10 for faster convergence.

The experimental basis discussed here to obtain the response function is the impulse test. An impulse load input of known peak value is applied for a very short time duration to the system whose response function is sought. Both the impulse input force and the output deflections are

recorded as functions of time. Figure 5 shows schematic traces of the input and the output necessary to determine such a response function in conjunction with Equation 13. The computation results in vector quantities of the response function. These values, shich are spaced Δt apart in the time domain, can then be approximated by a mathematical function using a non-linear least square curve fit method. The closer the approximation to the actual response function curve, the better becomes the approximation of the response function sought. The function used in this investigation is

$$G(t) = \alpha e^{-\beta t} \sin \gamma t \tag{14}$$

where $G(t)$ = response function,

t = time, and

α, β and γ are constants.

As a check on the response functions prior to curve fitting, the finite difference solution of Equation 12 was obtained explicitly [64] as

$$O(t_k) = \sum_{j=1}^{k} G(t_j)\ I(t_{k-j+1})\ \Delta t \tag{15}$$

Thus, the output calculated from Equation 15 should match the measured output used to obtain the response function.

The modified version of the computer program [64] used to calculate the response functions and to predict the output is shown in Appendix A. Subroutines CONVLI and CONVLE were employed to solve, respectively, Equation 13 and Equation 15.

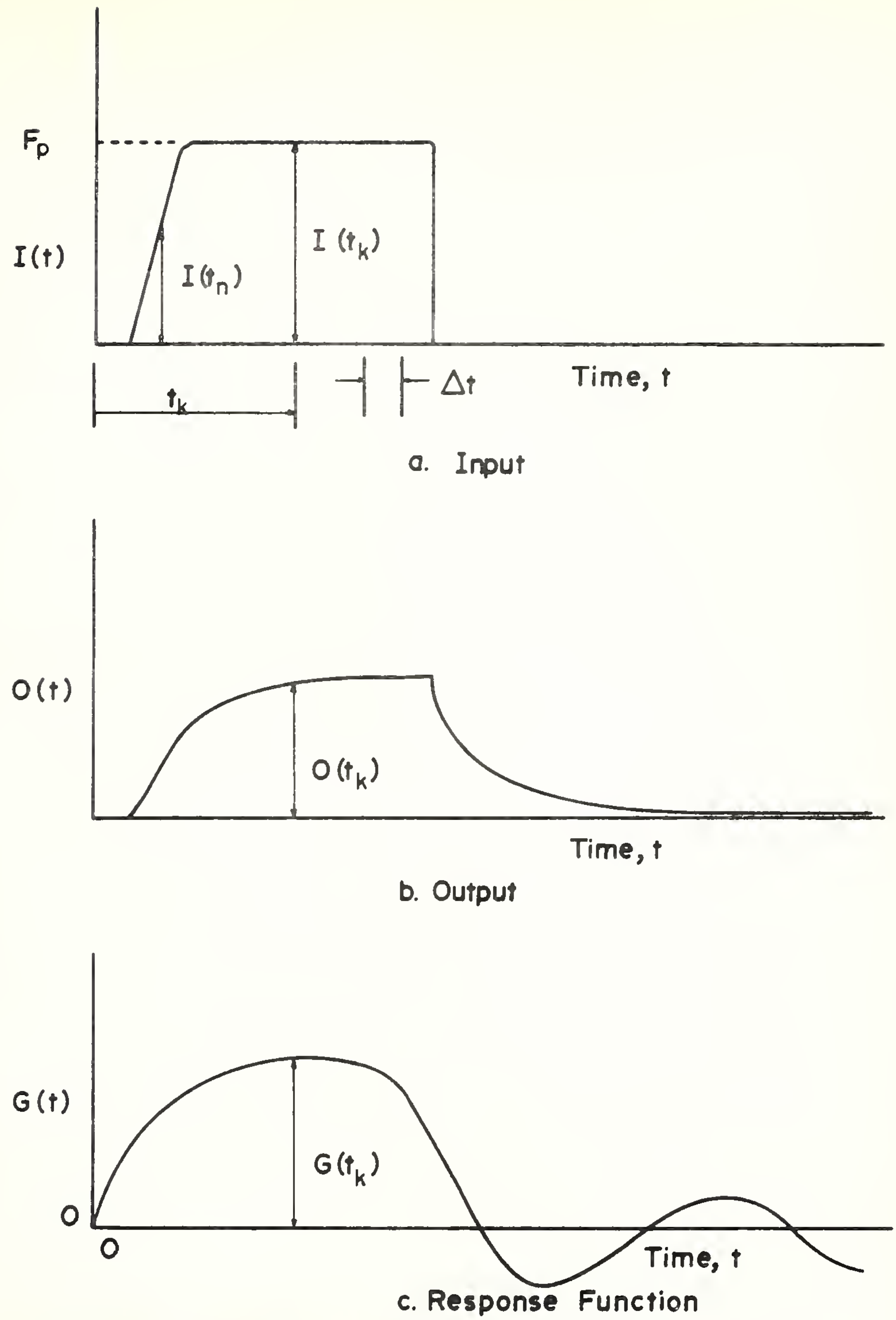


FIGURE 5 EXPERIMENTAL DETERMINATION OF SYSTEM RESPONSE FUNCTION

Prediction Theory

If the system represented by Equation 9 (or Equation 14) is subjected to an excitation force F(t) and the corresponding output time function is y(t), the transformed response is

$$\bar{y}(s) = G(s)\ \bar{F}(s) \tag{16}$$

By the convolution theorem,

$$\begin{aligned} y(t) &= L^{-1}[G(s)\ \bar{F}(s)] \\ &= G(t)*F(t) \\ &= \int_0^t G(\tau)\ F(t-\tau)d\tau \end{aligned} \tag{17}$$

Equation 17 illustrates that one can calculate the system response to any excitation function F(t) if the response function G(t) is known. When the input function has a mathematical representation, the convolution Equation 17 can be solved analytically. The solutions for a step load input and a repeated load input are derived below for a pavement system whose response function is G(t).

Solution for Step Loading

Consider the step function input F(t) defined as

$$\begin{aligned} F(t) &= 0 \qquad \text{for } t < 0 \\ F(t) &= F_o \qquad \text{for } t \geq 0 \end{aligned} \tag{18}$$

where F_o is the magnitude of the static compressive force. However, due to the introduction of the conversion factor K

in obtaining the response functions, Equation 18 has to be modified as

$$
\begin{aligned}
F(t) &= 0 && \text{for } t < 0 \\
F(t) &= KF_o && \text{for } 0 \leq t \leq t_n \\
F(t) &= F_o && \text{for } t > t_n
\end{aligned}
\qquad (19)
$$

where t_n is the time at which the input, from which the response function is obtained, first becomes non-zero. Substituting Equations 14 and 19 into Equation 17,

$$
\begin{aligned}
y(t) &= KF_o \int_0^{t_n} \alpha e^{-\beta\tau} \sin \gamma\tau \, d\tau + F_o \int_{t_n}^{t} \alpha e^{-\beta\tau} \sin \gamma\tau \, d\tau \\
&= F_o \, \alpha \left[\frac{K\gamma}{\beta^2 + \gamma^2} - \frac{(K-1)}{\sqrt{\beta^2 + \gamma^2}} e^{-\beta t_n} \sin (\gamma t_n + \phi) \right. \\
&\qquad \left. - \frac{e^{-\beta t}}{\sqrt{\beta^2 + \gamma^2}} \sin (\gamma t + \phi) \right]
\end{aligned}
\qquad (20)
$$

For $t < t_n$,

$$
\begin{aligned}
y(t) &= KF_o \int_0^{t} \alpha e^{-\beta\tau} \sin \gamma\tau \, d\tau \\
&= KF_o \, \alpha \left[\frac{\gamma}{\beta^2 + \gamma^2} - \frac{e^{-\beta t}}{\sqrt{\beta^2 + \gamma^2}} \sin (\gamma t + \phi) \right]
\end{aligned}
\qquad (20a)
$$

where $\phi = \tan^{-1}(\gamma/\beta)$ (21)

At $t = 0$, Equation 20a yields:

$$y(0) = 0 \qquad (22)$$

As t becomes large, the steady-state response is obtained from Equation 20 as

$$y(t) = F_o \alpha \left[\frac{K\gamma}{\beta^2 + \gamma^2} - \frac{(K-1)}{\sqrt{\beta^2 + \gamma^2}} e^{-\beta t_n} \sin(\gamma t_n + \phi) \right] \quad (23)$$

The computer program STALOD to solve Equation 23 is given in Appendix B.

Solution for Repeated Loading

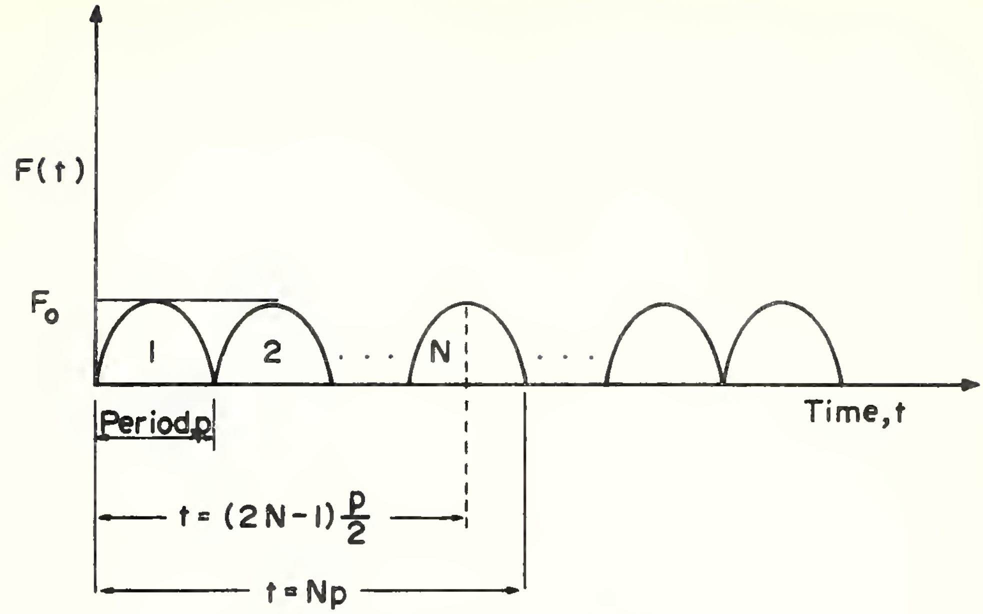
With the modification discussed in the previous section taken into account, a haversine repeated load configuration shown in Figure 6 is defined as

$$\begin{aligned} F(t) &= 0 && \text{for } t < 0 \\ F(t) &= \frac{KF_o}{2}(1 - \cos \omega t) && \text{for } 0 \leq t \leq t_n \\ F(t) &= \frac{F_o}{2}(1 - \cos \omega t) && \text{for } t > t_n \end{aligned} \quad (24)$$

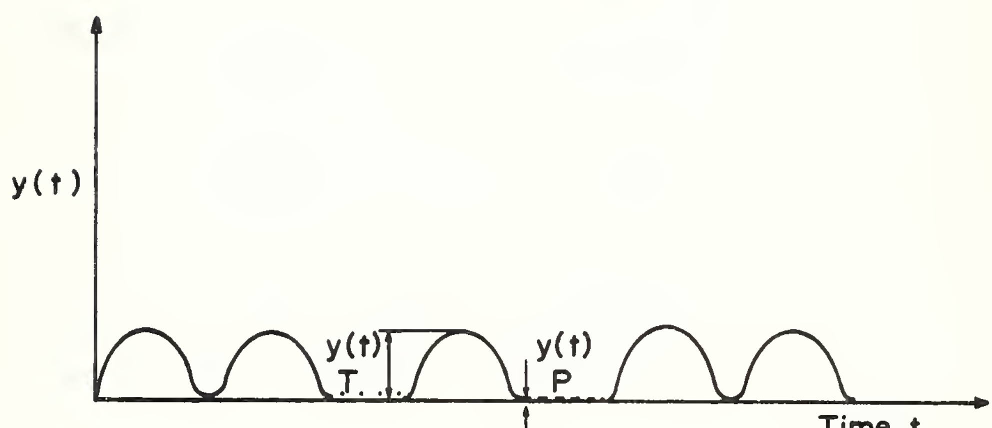
where $\omega = 2\pi/p$, in which p = period of loading.

Substituting Equations 14 and 24 into Equation 17,

$$\begin{aligned} y(t) &= \frac{KF_o}{2} \int_0^{t_n} \alpha e^{-\beta\tau} \sin \gamma\tau \, (1 - \cos \omega(t-\tau)) d\tau \\ &+ \frac{F_o}{2} \int_{t_n}^{t} \alpha e^{-\beta\tau} \sin \gamma\tau \, (1 - \cos \omega(t-\tau)) d\tau \\ &= \frac{F_o \alpha}{2} \left[\frac{K\gamma}{\beta^2 + \gamma^2} - \frac{(K-1)}{\sqrt{\beta^2 + \gamma^2}} e^{-\beta t_n} \sin(\gamma t_n + \phi) \right. \\ &\left. - \frac{e^{-\beta t}}{\sqrt{\beta^2 + \gamma^2}} \sin(\gamma t + \phi) \right] \\ &- \frac{F_o \alpha}{4} \cos \omega t \left\{ K \left[\frac{(\gamma - \omega)}{\beta^2 + (\gamma - \omega)^2} + \frac{(\gamma + \omega)}{\beta^2 + (\gamma + \omega)^2} \right] \right. \end{aligned}$$



a. Input: $F(t) = \frac{F_o}{2}\left(1 - \cos\frac{2\pi}{p}t\right)$



b. Output

FIGURE 6 HAVERSINE INPUT AND OUTPUT FUNCTIONS

$$
\begin{aligned}
&- (K-1)\left[\frac{e^{-\beta t_n}}{\sqrt{\beta^2 + (\gamma-\omega)^2}} \sin\left((\gamma-\omega)t_n + \phi_1\right)\right. \\
&+ \left.\frac{e^{-\beta t_n}}{\sqrt{\beta^2 + (\gamma+\omega)^2}} \sin\left((\gamma+\omega)t_n + \phi_2\right)\right] \\
&- \frac{e^{-\beta t}}{\sqrt{\beta^2 + (\gamma-\omega)^2}} \sin\left((\gamma-\omega)t + \phi_1\right) \\
&- \left.\frac{e^{-\beta t}}{\sqrt{\beta^2 + (\gamma+\omega)^2}} \sin\left((\gamma+\omega)t + \phi_2\right)\right\} \\
&- \frac{F_o\alpha}{4} \sin \omega t \left\{K\left[\frac{\beta}{\beta^2 + (\gamma-\omega)^2} - \frac{\beta}{\beta^2 + (\gamma+\omega)^2}\right]\right. \\
&- (K-1)\left[\frac{e^{-\beta t_n}}{\sqrt{\beta^2 + (\gamma-\omega)^2}} \cos\left((\gamma-\omega)t_n + \phi_1\right)\right. \\
&- \left.\frac{e^{-\beta t_n}}{\sqrt{\beta^2 + (\gamma+\omega)^2}} \cos\left((\gamma+\omega)t_n + \phi_2\right)\right] \\
&- \frac{e^{-\beta t}}{\sqrt{\beta^2 + (\gamma-\omega)^2}} \cos\left((\gamma-\omega)t + \phi_1\right) \\
&- \left.\frac{e^{-\beta t}}{\sqrt{\beta^2 + (\gamma+\omega)^2}} \cos\left((\gamma+\omega)t + \phi_2\right)\right\} \qquad (25)
\end{aligned}
$$

where $\phi = \tan^{-1} (\gamma/\beta)$,

$$\phi_1 = \tan^{-1}\left(\frac{\gamma-\omega}{\beta}\right), \text{ and}$$

$$\phi_2 = \tan^{-1}\left(\frac{\gamma+\omega}{\beta}\right). \qquad (26)$$

<u>Total</u> <u>Response</u>. The total response $y_T(t)$ of the system is evaluated when a load cycle is at its peak position (Figure 6), that is when

$$t = (2N-1)\ \frac{p}{2} \tag{27}$$

where t = time,

N = number of load applications, and

p = period of loading.

Since $\sin \omega t = \sin\left(\frac{2\pi}{p}\ (2N-1)\ \frac{p}{2}\right) = 0,$

and $\cos \omega t = \cos\left(\frac{2\pi}{p}\ (2N-1)\ \frac{p}{2}\right) = -1,$

Equation 26 reduces to

$$\begin{aligned}
y_T(t) = \frac{F_o\alpha}{2} \Bigg\{ & \frac{K\gamma}{\beta^2+\gamma^2} + \frac{K}{2}\left[\frac{\gamma-\omega}{\beta^2+(\gamma-\omega)^2} + \frac{\gamma+\omega}{\beta^2+(\gamma+\omega)^2}\right] \\
& - \frac{(K-1)}{\sqrt{\beta^2+\gamma^2}}\ e^{-\beta t_n} \sin\ (\gamma t_n + \phi) \\
& - \left(\frac{K-1}{2}\right)\left[\frac{e^{-\beta t_n}}{\sqrt{\beta^2+(\gamma-\omega)^2}} \sin\left((\gamma-\omega)t_n + \phi_1\right)\right. \\
& \left. + \frac{e^{-\beta t_n}}{\sqrt{\beta^2+(\gamma+\omega)^2}} \sin\left((\gamma+\omega)t_n + \phi_2\right)\right] \\
& - \frac{e^{-\beta t}}{\sqrt{\beta^2+\gamma^2}} \sin\ (\gamma t + \phi) \\
& - \frac{1}{2}\left[\frac{e^{-\beta t}}{\sqrt{\beta^2+(\gamma-\omega)^2}} \sin\left((\gamma-\omega)t + \phi_1\right)\right. \\
& \left. + \frac{e^{-\beta t}}{\sqrt{\beta^2+(\gamma+\omega)^2}} \sin\left((\gamma+\omega)t + \phi_2\right)\right]\Bigg\}
\end{aligned} \tag{28}$$

<u>Permanent or Accumulative Response</u>. The permanent response $y_p(t)$ is measured at the end of a load cycle (Figure 6), that is when

$$t = Np \tag{29}$$

Since $\sin \omega t = \sin\left(\frac{2\pi}{p} Np\right) = 0$

and $\cos \omega t = \cos\left(\frac{2\pi}{p} Np\right) = 1,$

Equation 26 yields

$$\begin{aligned}
y_p(t) = \frac{F_o\alpha}{2} \Bigg\{ & \frac{K\gamma}{\beta^2+\gamma^2} - \frac{K}{2}\left[\frac{\gamma-\omega}{\beta^2+(\gamma-\omega)^2} + \frac{\gamma+\omega}{\beta^2+(\gamma+\omega)^2}\right] \\
& - \frac{(K-1)}{\sqrt{\beta^2+\gamma^2}}\, e^{-\beta t_n} \sin(\gamma t_n + \phi) \\
& + \left(\frac{K-1}{2}\right)\left[\frac{e^{-\beta t_n}}{\sqrt{\beta^2+(\gamma-\omega)^2}} \sin\left((\gamma-\omega)t_n + \phi_1\right)\right. \\
& \left. + \frac{e^{-\beta t_n}}{\sqrt{\beta^2+(\gamma+\omega)^2}} \sin\left((\gamma+\omega)t_n + \phi_2\right)\right] \\
& - \frac{e^{-\beta t}}{\sqrt{\beta^2+\gamma^2}} \sin(\gamma t + \phi) \\
& + \frac{1}{2}\left[\frac{e^{-\beta t}}{\sqrt{\beta^2+(\gamma-\omega)^2}} \sin\left((\gamma-\omega)t + \phi_1\right)\right. \\
& \left. + \frac{e^{-\beta t}}{\sqrt{\beta^2+(\gamma+\omega)^2}} \sin\left((\gamma+\omega)t + \phi_2\right)\right]\Bigg\}
\end{aligned} \tag{30}$$

Appendix C presents the computer program REPLOD to solve Equations 28 and 30.

EXPERIMENTAL INVESTIGATION

The experimental phase of the study was undertaken to investigate the theory of pavement behavior developed in the preceding section. The scope of the investigation, materials selected, design and preparation of the pavement models, instrumentation and test procedures are described.

Scope

The study was limited to controlled laboratory testing of three-layer flexible model pavements constructed of representative paving materials. A silty sand subgrade, a crushed aggregate base, and an asphalt concrete surface were the components of the three-layer systems. Surface course thicknesses of 1 and 2 inches were used. The $3\frac{1}{2}$ - inch base course thickness was estimated by an approximate design procedure briefly outlined in the next subsection. The use of an asphalt-treated base and a 3-inch surface course thickness originally planned to be considered were deleted because the first case amounts to increasing the surface course thickness over the untreated base [3, 60], while in the latter case, only the response of the top layers would have been measured.

Static loads and two dynamic loads, namely, impulse and cyclic loadings were thought to simulate the loading conditions which occur in a prototype pavement. Contact pressures of 14.93, 29.86 and 59.72 psi were applied over a circular loaded area of 4-inch diameter at the center of the pavements. These stress magnitudes were considered representative of those encountered in highway pavements by vehicle wheels. The loading plate used was chosen to be small relative to the pavement surface, but yet to allow a sufficient portion of the subgrade to be stressed. The tests were carried out under isothermal conditions in a constant temperature room at 50, 75 and 100°F.

The model pavement with 2-inch surfacing was also subjected to corner loadings to study the edge effects that might result as a consequence of the sides of the box. Loading modes were impulse and cyclic. The test temperature was 50°F for the corner loadings since any edge effects would be more pronounced, if any were to be significant, at these test conditions. Only the largest stress magnitude, namely, 59.72 psi was applied over the 4-inch diameter plate for the edge loading condition.

Approximate Design of the Model Pavements

Influence curves developed by Nijboer [65], from the three-layer elastic analysis presented by Jones [13], in conjunction with the equivalent layer theory [66], were used for the purpose of estimating the thickness of the

base course. A brief outline of the procedure follows:

1. From data given by Yoder [6], the elastic moduli $E_3 = 2{,}500$ psi and $E_2 = 5{,}000$ psi, were obtained for the subgrade and the base course, respectively. The corresponding value for the surface mixture was $E_1 = 40{,}000$ psi [15, 57].

2. With a limiting deflection of 0.025 in. [67, 68], an applied pressure of 80 psi, a radius of contact area of 2 in., the influence curve [65, Graph IV] yields a thickness of 2.7 in. of asphaltic concrete over the subgrade.

3. A one-inch surface was thought reasonable.

4. The remaining 1.7 in. multiplied by the approximate surface course/base course equivalency of $\sqrt[3]{\frac{E_1}{E_2}} = 2$ gave, upon rounding to the nearest half-inch, $3\frac{1}{2}$ in. of base course.

Materials and Preparation of Model Pavements

The model pavements were contained in a wooden box reinforced with steel angles. Since the interest was in testing the pavement models inside a constant temperature room, the maximum possible size of the box adopted had the internal dimensions $32\frac{1}{2}$ by $32\frac{1}{2}$ by $23\frac{1}{4}$ inches deep. To minimize moisture losses, the box was made of exterior type plywood and the interior sides and bottom of the box were coated with spar varnish diluted with linseed oil.

The next three sections present the results of tests performed on each of the materials, followed by a description of placing the respective material in the box.

Subgrade Soil

A silty sand was used for the subgrade. Table 1 summarizes the physical properties and Figure 7 shows the grain size distribution curve of the sand. Compaction test results are given in Figure 8.

The soil was stored in air-tight bags at its natural moisture content of 19.1 percent. A sufficient amount of soil to make a compacted lift of about one inch was then spread out in large trays. The clods were broken by hand and the soil was mixed frequently to insure uniform drying. Periodically, the moisture content was determined using a Speedy Moisture Tester, shown in Figure 9, which was pre-calibrated for this particular soil (Figure 10). When the moisture content was about 0.5 percent above the optimum, the soil was placed in the box in a predetermined amount necessary per lift. The soil was then compacted in the box using an air hammer, weighing 14 pounds, with a 3-inch diameter base. The air supply was set at 75 psi. Compaction at the edges and leveling were achieved by a hand tamp weighing 16 pounds and with a base plate 8 inches square. Figure 11 shows the compaction process. Marked lines on the interior of the box sides indicated when the required compaction was obtained as calculated from Equation 31:

$$\gamma_{wet} = \frac{CW}{h} \tag{31}$$

TABLE 1

PHYSICAL PROPERTIES OF THE SUBGRADE SOIL

Classification, Unified System - - - - - - - - - -	SM
Liquid Limit,[1] % - - - - - - - - - - - - - - - - - -	29.3
Plastic Limit,[2] % - - - - - - - - - - - - - - - - -	23.3
Plasticity Index,[2] % - - - - - - - - - - - - - - - -	6.0
Optimum Moisture Content,[3] % - - - - - - - - - - -	13.7
Optimum Dry Density,[3] lbs./ft^3 - - - - - - - - - -	114.4
Specific Gravity[4] - - - - - - - - - - - - - - - - - -	2.73
Soaked CBR (California Bearing Ratio),[5] % - - -	10.0

[1] AASHO T 89

[2] AASHO T 90

[3] AASHO T 99

[4] AASHO T 100

[5] AASHO T 193

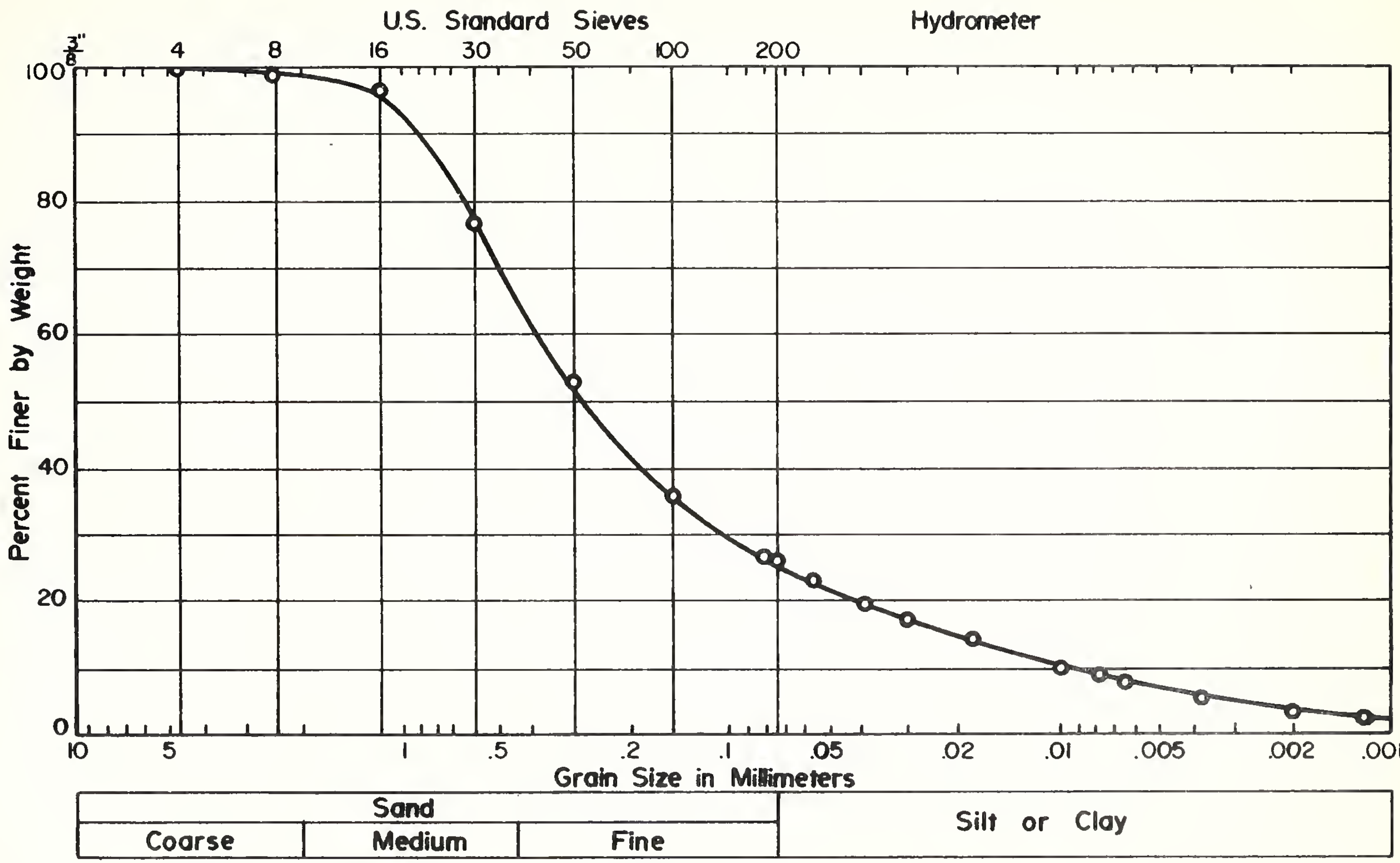


FIGURE 7 GRAIN SIZE DISTRIBUTION CURVE FOR THE SUBGRADE SOIL

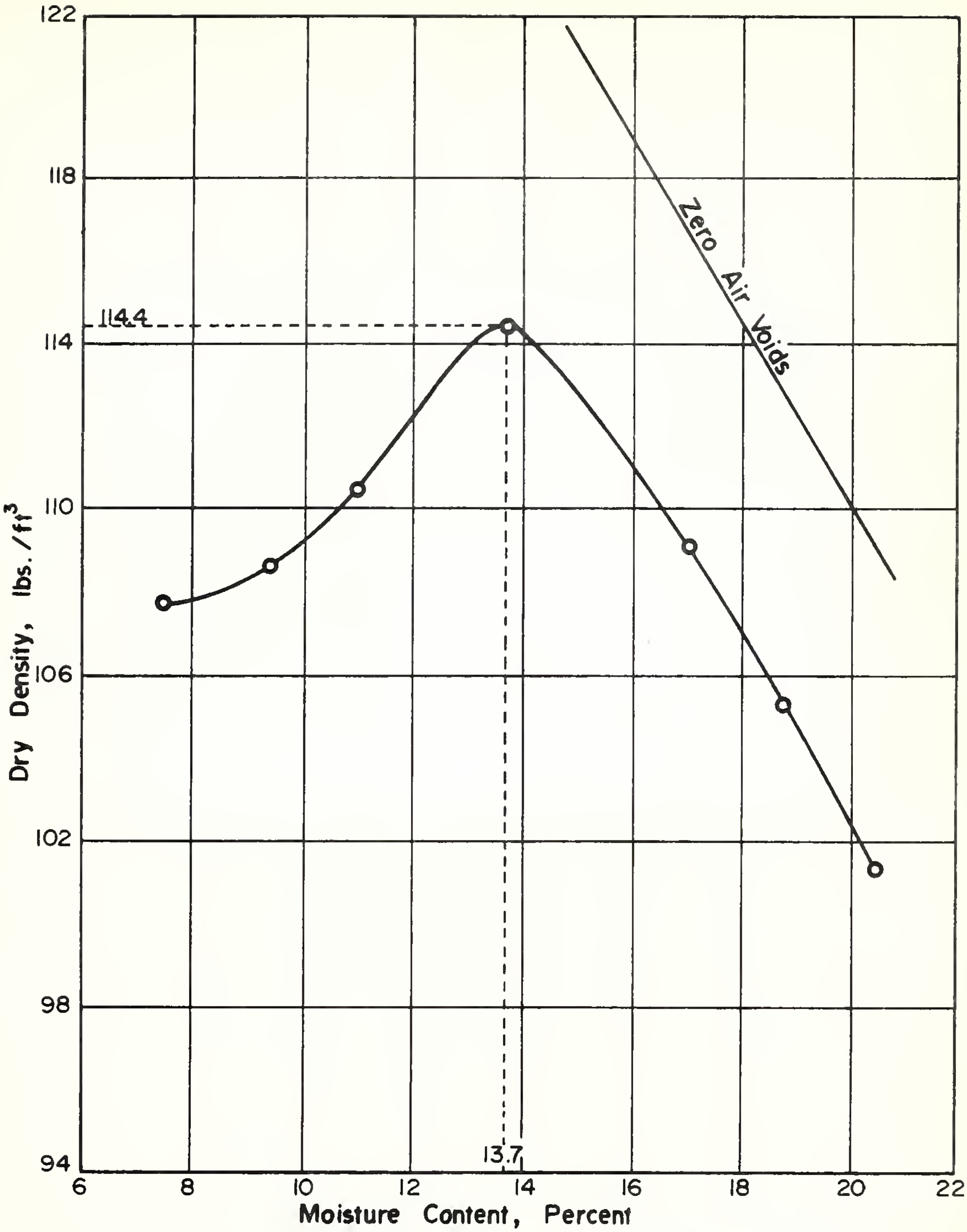


FIGURE 8 MOISTURE-DENSITY RELATIONSHIP OF THE SUBGRADE SOIL

FIGURE 9 THE SPEEDY MOISTURE TESTER

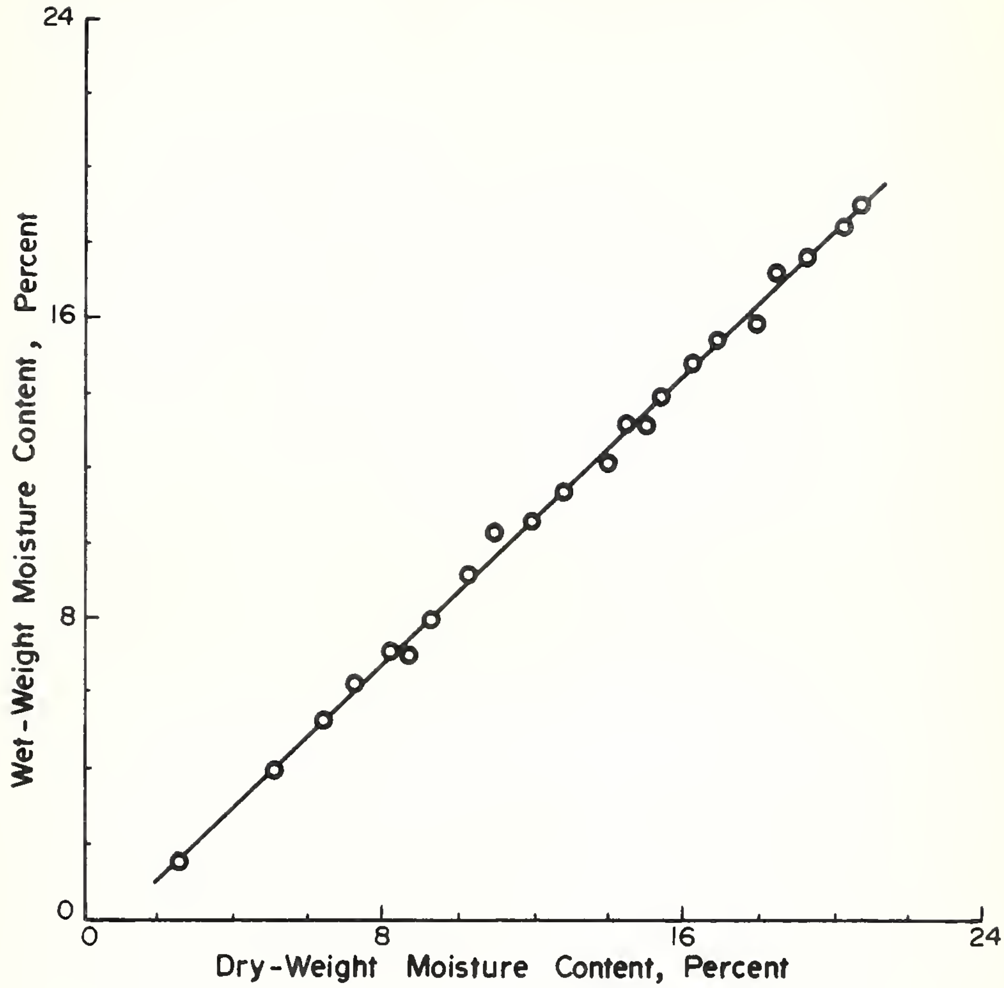


FIGURE 10 CALIBRATION OF THE SPEEDY MOISTURE TESTER FOR THE SUBGRADE SOIL

FIGURE 11 SUBGRADE SOIL UNDER COMPACTION

where γ_{wet} = wet density of soil in $lbs./ft^3$,

W = weight of soil layer in pounds,

h = compacted thickness of layer in inches.

The constant C was calculated from the geometry of the box to be 1.64.

Before placing each subsequent layer, the smooth surface of the compacted soil in the box was scarified to minimize compaction planes. When a compacted thickness of about 4 inches was obtained, in-situ density was checked by the sand-cone method (AASHO T 191). Samples were taken at each in-place density test for water content determination.

The subgrade compaction was completed in 20 layers to give a total compacted thickness of $16\frac{3}{4}$ inches. The average dry density of the compacted lifts was 115.5 lbs. per cu. ft. which was 101% of the standard AASHO T 99. The moisture content averaged 13.9%.

Base Course

The material selected for the base course was a crushed limestone aggregate conforming with Indiana specification Size No. 53 (Figure 12). Tests performed according to AASHO T 99 and AASHO T 193 gave, respectively, an optimum density of 126.4 $lbs./ft^3$ at an optimum moisture content of 6.2%, and a soaked CBR of 51.3 percent.

The aggregate was compacted to the total of $3\frac{1}{2}$ inches thick, in three lifts, with the same air hammer used for the subgrade. However, for the base course, the circular base

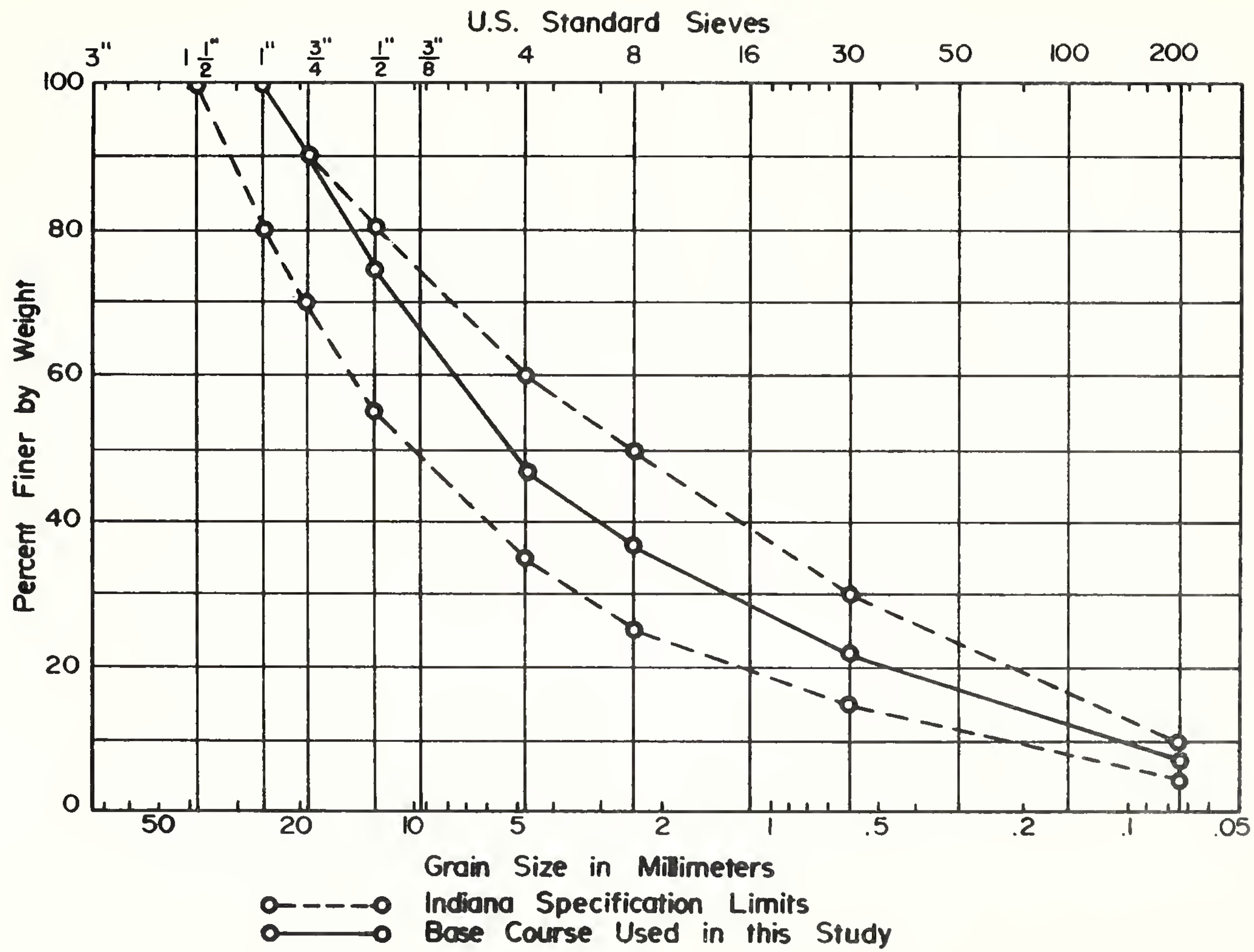


FIGURE 12 GRADATION CURVE FOR INDIANA SIZE NO. 53 BASE COURSE

plate of the hammer was replaced with a $3\frac{1}{2}$-inch square one. Leveling of the surface was accomplished by a vibrator. A 6-inch by 3-inch base plate attached to the vibrator served as a leveling foot. Both the air hammer and the vibrator were operated at a line pressure of 75 psi. The base course was compacted to 100% of the maximum dry density (AASHO T 99) as controlled by Equation 31.

Surface Course

The bituminous mixture used for the surface course was prepared from crushed limestone and sand aggregate blended to conform with Indiana type B surface mixture gradation. Sieve analysis and origin of the aggregate are given in Table 2. Figure 13 shows the gradation range for Indiana Type B Surface Mixtures and the gradation chosen for this investigation.

A 60-70 penetration asphalt cement having the properties shown in Table 3 was used for the surface course. The weight of asphalt cement mixed with the aggregate was 6 percent by weight of aggregate in accordance with Indiana specifications.* Specimens were made and tested according to the Hveem method of mix design [69]. The Hveem test results are summarized below:

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* In the specifications, the range of asphalt content for Type B Surface Mixture is 5.5 - 6.6% by weight of mix.

TABLE 2

SIEVE ANALYSIS OF AGGREGATES FOR

THE SURFACE COURSE MIXTURE

Sieve Size Passing	Sieve Size Retained	Fraction %	Material
1/2 in.	3/8 in.	14	Crushed Delphi Limestone
3/8 in.	No. 4	36	" " "
No. 4	No. 8	11	" " "
No. 8	No. 16	12	Lafayette River Sand
No. 16	No. 30	12	" " "
No. 30	No. 50	7	" " "
No. 50	No. 100	4	" " "
No. 100	No. 200	2	" " "
No. 200		2	Greencastle Limestone Filler

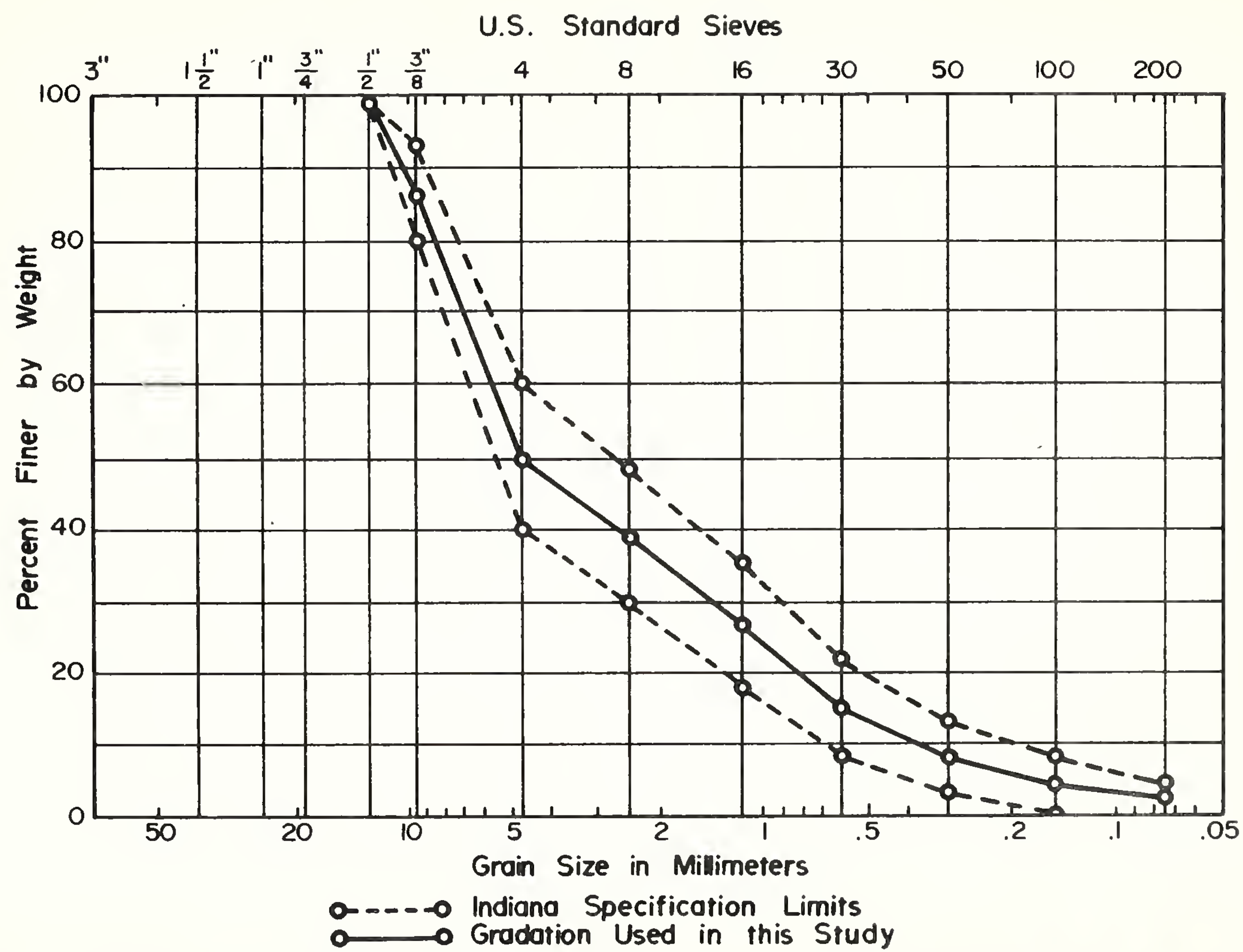

FIGURE 13 GRADATION CURVE FOR INDIANA TYPE B SURFACE MIXTURE

TABLE 3

RESULTS OF TESTS ON ASPHALT CEMENT

Penetration, 100 grams, 5 sec., 77°F[1] - - - - - -	62
Penetration, 200 grams, 60 sec., 32°F[1] - - - - - -	16
Loss on Heating, 50 grams, 5 hr., 325°F,[2] % - - -	0.02
Penetration of Residue,[1,2] % of Original - - - -	83
Softening Point, Ring and Ball,[3] °F - - - - - - -	123
Specific Gravity at 77°F [4] - - - - - - - - - - - -	1.011
Flash Point, Cleveland Open Cup,[5] °F - - - - - - -	581
Ductility at 77°F, 5 cm/min.,[6] cm - - - - - - - -	150+
Kinematic Viscosity at 275°F,[7] cSt - - - - - - - -	460
Kinematic Viscosity at 325°F,[7] cSt - - - - - - - -	137

[1] ASTM D 5

[2] ASTM D 6

[3] ASTM D 36

[4] ASTM D 70

[5] ASTM D 92

[6] ASTM D 113

[7] ASTM D 2170

Specimen	Unit Weight ($lbs./ft^3$)	Hveem Stability	Air Voids (%)
A60-1	150.72	36.8	3.66
A60-2	151.01	57.8	3.85
A60-3	149.82	49.7	5.07
Average	150.52	48.1	4.19

Prior to preparing the bituminous mixture to be used as surface course for the pavement models, compaction and leveling experimentation was conducted with the air hammer and vibrator. Although the devices proved to be satisfactory in placing the base course, it was thought essential to correlate, by trial, the time of compaction and leveling with the compactive effort required to obtain the design density at the selected asphalt content.

The mixture for the surface course was prepared by weighing 5000-gram batches of total aggregate, heating the aggregate to 325°F and then mixing it with the asphalt, heated to the same temperature, in a mechanical mixer for two minutes. After mixing was completed, each batch was put in a flat pan and placed in a forced-air draft oven to cure for 15 hours at 140°F.

To minimize the temperature loss from the mixture due to air drifts from the air hammer during compaction, the constant temperature room was set to the maximum operating temperature of 113°F. Compaction of the bituminous mixture

was started after the mixture was reheated to 325°F. The hot mixture was spread uniformly over the base course in one layer such that the compacted thickness would be one inch. The sides of the box served as the form for laying and compacting the mixture. The bituminous mixture was compacted and leveled by the heated air hammer and vibrator for the predetermined time. The density obtained for the one-inch asphaltic concrete surfacing was 149.8 pcf. The in-place void content corresponding to this density was calculated to be 4.6 percent.

Upon completion of the tests with the model pavement having a one-inch surfacing, more batches of bituminous mixture to increase the surfacing thickness to two inches were prepared in the above manner. A wooden form, with $32\frac{1}{2}$ by $32\frac{1}{2}$ by $3\frac{1}{2}$ inches internal dimensions connected to the top of the box was used during spreading and compaction of the mixture over the one-inch asphalt surface. Figure 14 shows the bituminous mixture under compaction. Leveling of the surface by the vibrator is shown in Figure 15. The same compactive effort was applied to give the same density as for the one-inch surface.

Instrumentation

Instrumentation was carried out with consideration for recording the time-dependent input and output parameters. A crucial part of the instrumentation technique lies in devising a system which is capable of applying an impulse

FIGURE 14 BITUMINOUS MIXTURE UNDER COMPACTION

FIGURE 15 LEVELING OF THE SURFACE COURSE

force of a desired magnitude and duration, and of detecting the surface motion at any required location on the pavement surface.

The equipment consisted of a loading frame designed and fabricated of steel members coupled with an MTS electro-hydraulic actuator. Details of the loading frame and the rigid support for the model pavements are shown in Figure 16. A mechanical holder was made of methyl methacrylate and connected to the top plate (item 9) of the loading frame. The purpose was to hold the servoram while the MTS console was being programmed in tension prior to applying a single compressive square wave. The generation of an impulse by this technique is schematically illustrated in Figure 17.

Loads were applied by the MTS machine and measured with a 1000-lb. capacity Strainsert load cell, type FL1U, mounted between the hydraulic actuator and the loading plate. The output of the load cell was recorded with a Brush Mark 280 two-channel recorder.

Five Sanborn Linearsyn linear variable differential transformers (LVDT's) were used to sense the deflections. For the center loading, one LVDT, designated as no. 3, was located 3.25 inches from the center of the loading plate. The other four were spaced 2.5 inches apart along the same radial line as LVDT no. 3. Figure 18 shows the locations of the LVDT's during corner loading.

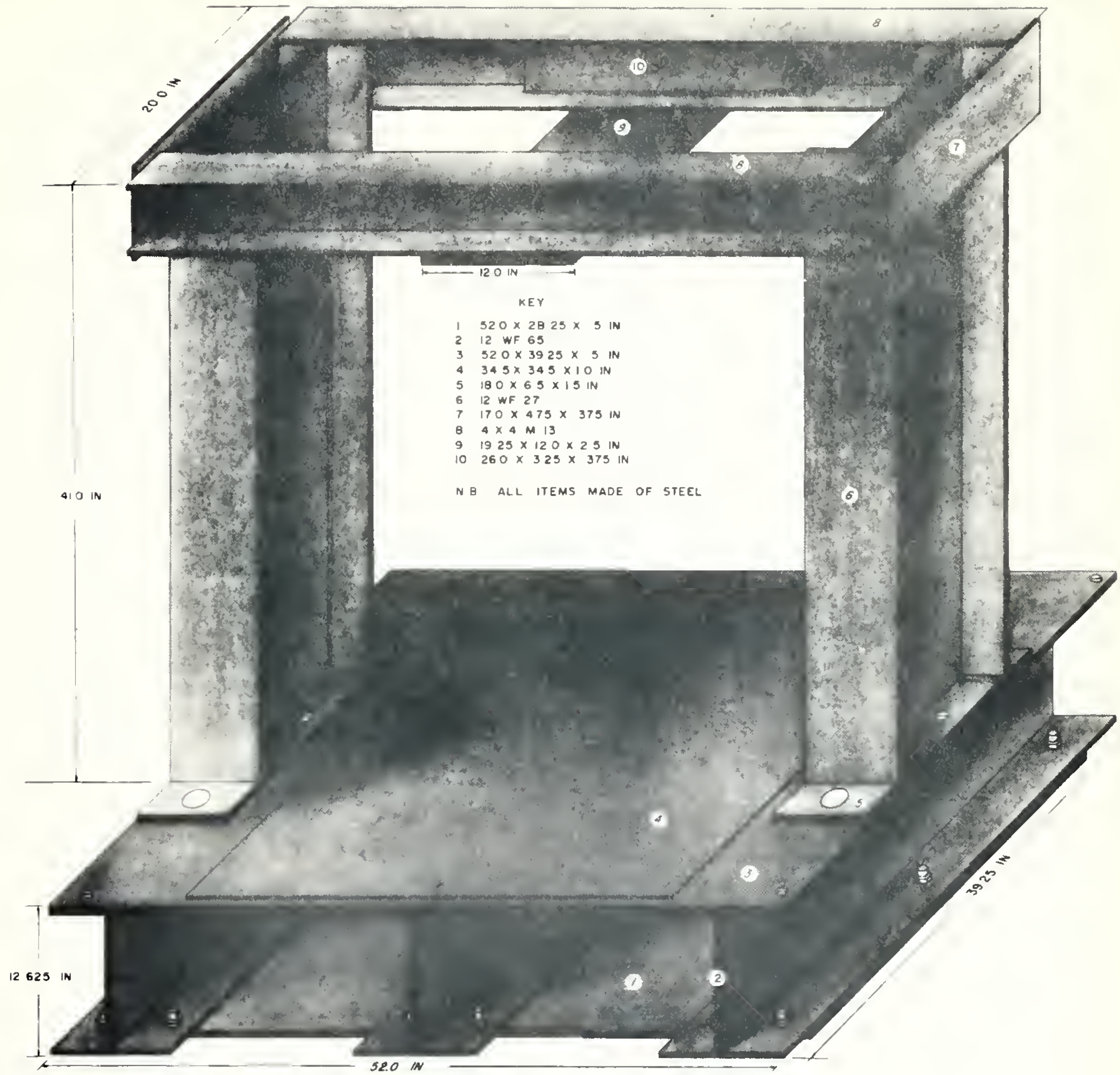

FIGURE 16 LOADING FRAME AND MODEL PAVEMENT SUPPORT

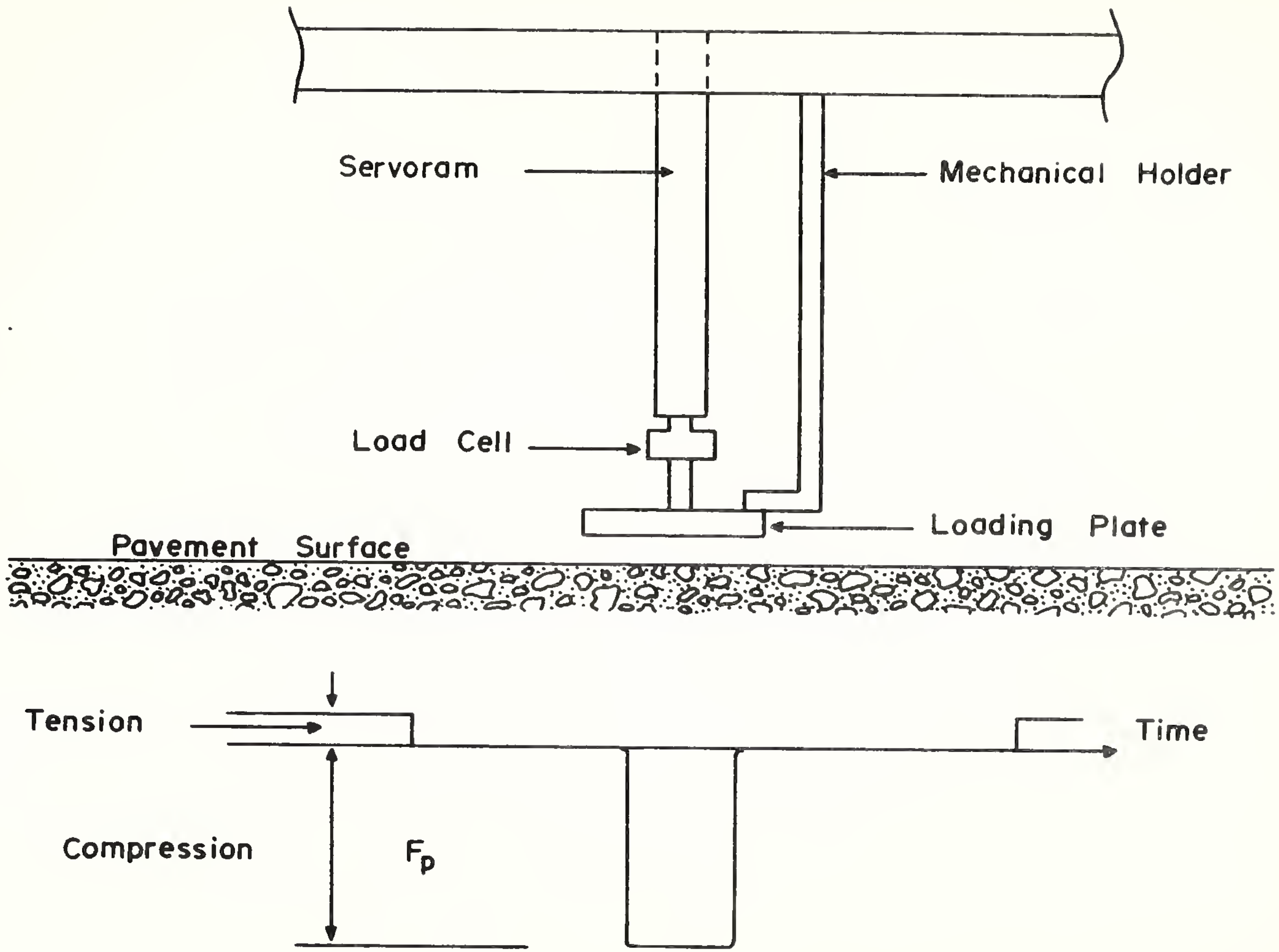

FIGURE 17 GENERATION OF IMPULSE

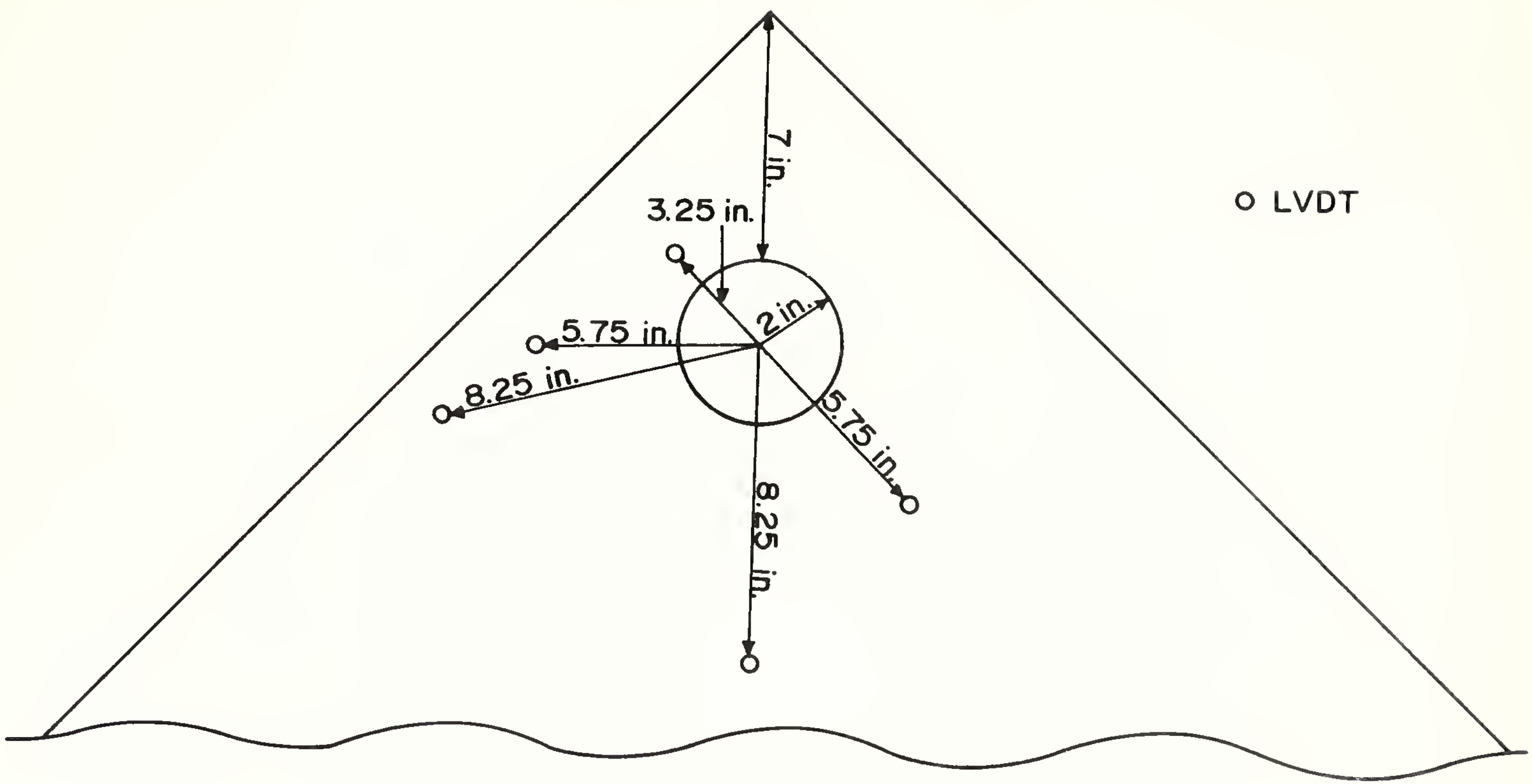

FIGURE 18 LOCATION OF PLATE AND LVTD'S: CORNER LOADING

Changes in output voltage from the LVDT's were amplified and recorded by an 8-channel Sanborn recorder. Calibration of the LVDT's was effected by displacing the core of the LVDT with a micrometer and relating this displacement to the magnitude of the pen deflection of the recorder. A typical calibration sheet for the LVDT's is shown in Table 4. One dial gage located at the same radial distance as LVDT no. 2 from the center of the load, but as close as possible to the LVDT, served as a check during static and cyclic testing.

The LVDT's were fixed to the pavement surface by screwing one end of a brass extension rod into the LVDT core and the other end into a $\frac{3}{4}$ - inch by $\frac{1}{2}$ - inch by $\frac{1}{8}$ - inch thick brass plate. The plate was in turn glued to the appropriate location on the pavement surface using Eastman 910 Adhesive. The LVDT's were supported by aluminum channels and plexiglas beams connected to the top angles of the box. Figure 19 shows the general view of the test set-up inside the constant temperature room for the center loading. A close view of the LVDT's is shown in Figure 20.

Test Procedures

Three tests are described in this section: impulse, static and repeated tests. The coding shown in Figure 21 was used to make data identification orderly.

TABLE 4

TYPICAL FORMAT FOR LVDT CALIBRATION

	Temperature:	75°F
	Power Amplifier Setting:	X·1
	Preamplifier Setting, mv/cm:	20
	Pen Displacement, mm:	20
Channel	LVDT Designation	LVDT Core Displacement, × 0.001 inch
1	1	3.0
2	2	3.2
6	3	3.0
7	4	3.2
8	5	2.8

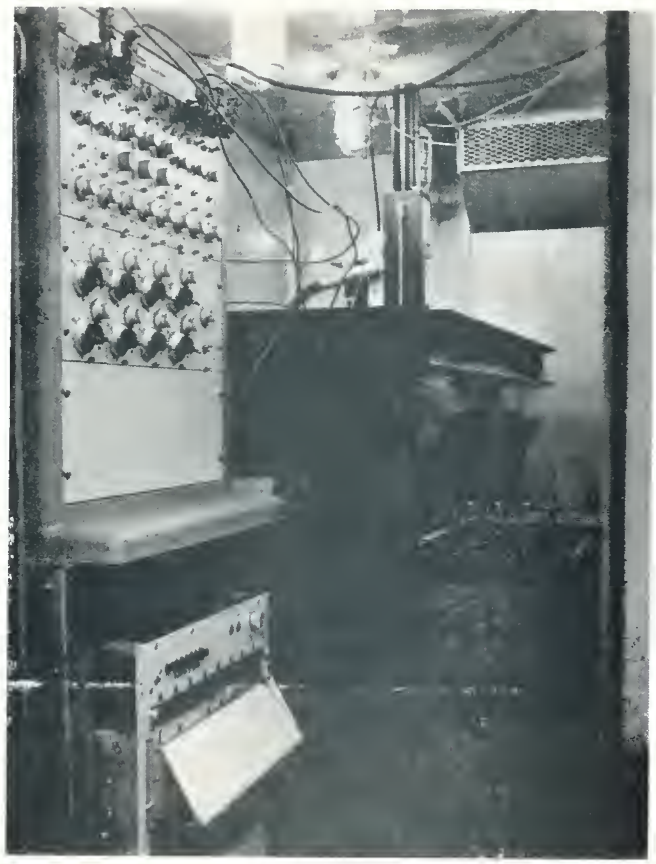

FIGURE 19 GENERAL VIEW OF TEST SETUP

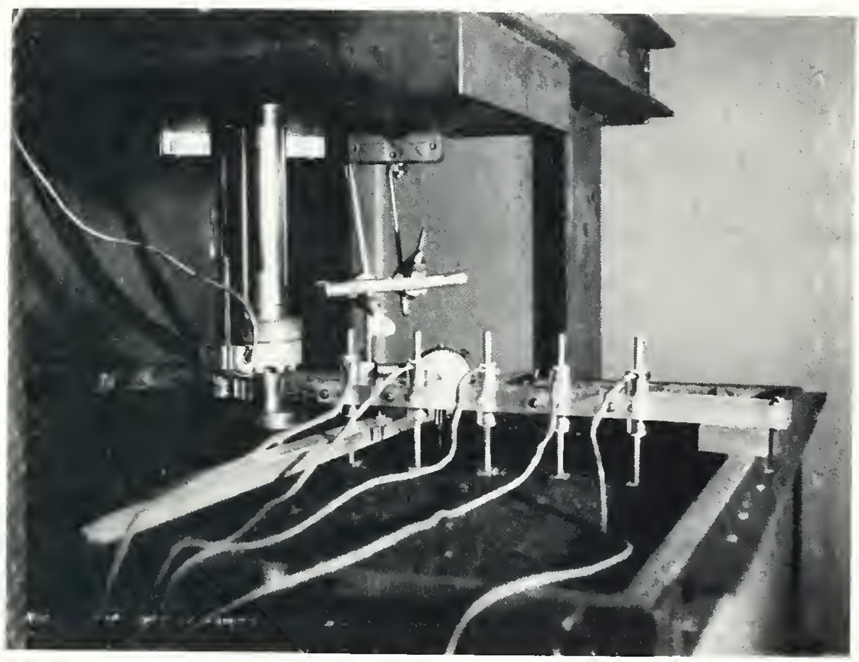

FIGURE 20 CLOSE VIEW OF TEST SETUP

Series I L I I

Load Intensity*:
14.93 psi (1), 29.86 psi (2), or 59.72 (3)

Input Function:
Impulse (I), Static (S), or Repeated (R)

Test Temperature:
50° F (L), 75° F (M), or 100° F (H)

Thickness of Surface Course:
1 inch (1), or 2 inches (2)

* Applied to a 4-inch diameter plate, these pressures amount to, respectively, 187.5, 375.0, and 750.0 lbs.

FIGURE 21 CODING OF DATA

Impulse Load Tests

With the function generator in the square-wave mode and by suitable manipulation of the MTS console, impulse load magnitudes of 187.5,* 375.0 and 750 lbs. were applied for a duration of about 0.16 sec. This time and a drop height of 0.5 inch were determined from preliminary tests during conditioning of the model pavements.

Typical traces of input load and output deflections are shown in Figure 22. A format for data reduction is illustrated in Table 5.

Static Load Tests

For static load tests, the frequency of the MTS machine was set at one cycle per second, the function generator in the ramp mode and the control mode in the manual trigger. By these adjustments, a static compressive force of the programmed magnitude was applied. The load was held on until the deflections became almost constant with time. This was found to be in the order of a few minutes.

Figure 23 shows representative graphical data for this type of test.

Repeated Load Tests

In advance of starting a cyclic load test, the function generator of the MTS machine was programmed in the compressive haversine form and in the control mode. Once testing started,

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* For Series 1HI1, the peak load was found to be 180.0 lbs.

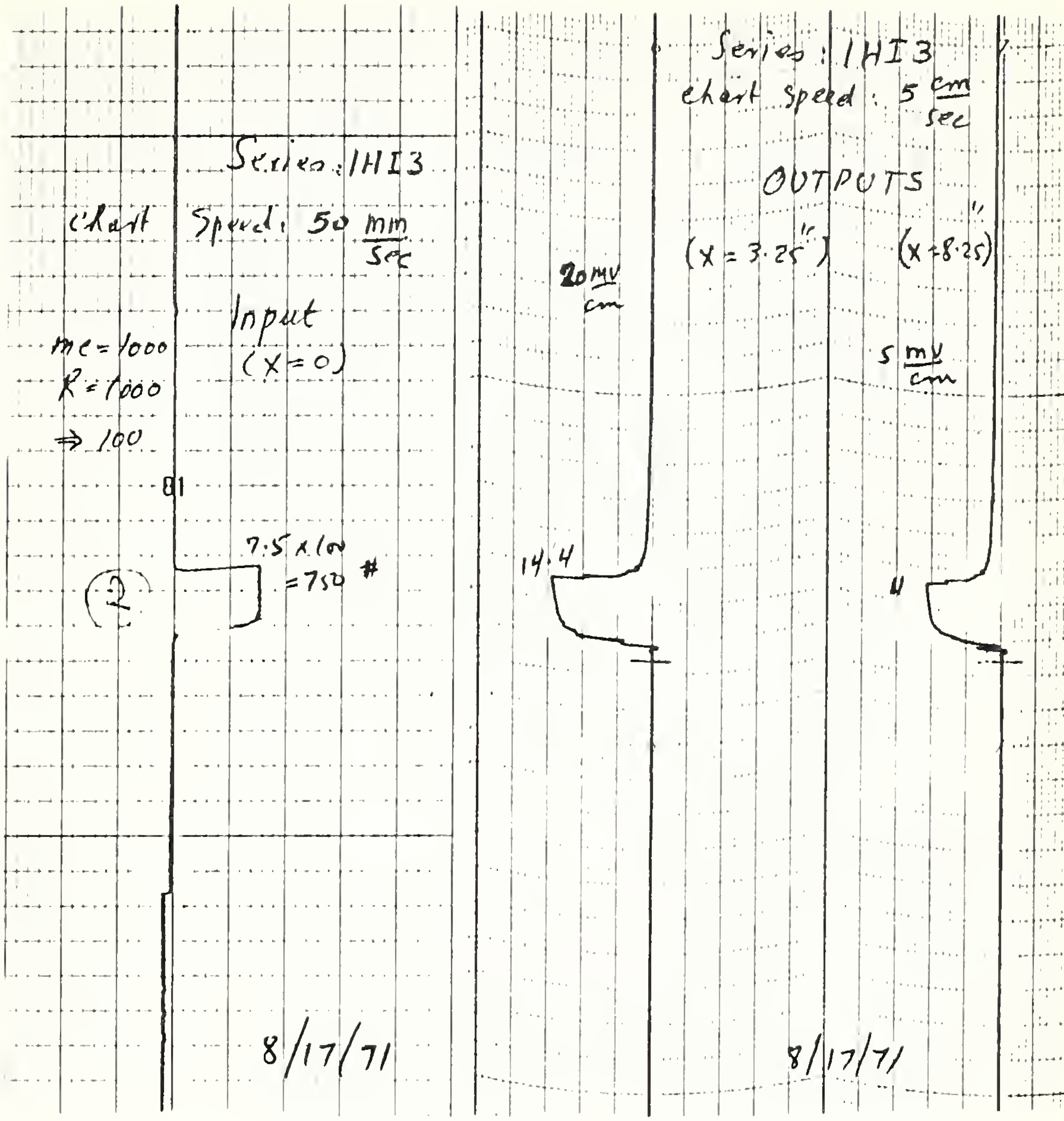

FIGURE 22 TYPICAL TRACES OF AN IMPULSE LOAD TEST

TABLE 5

TYPICAL DATA REDUCTION FOR IMPULSE TEST

Series: 1HI3

Time (sec.)	Distance from Load Center,* inches: 0		3.25		5.75		8.25	
	Pen Dispt. mm	Load lbs. ×100**	Pen Dispt. mm	Deflec. in.$\times 10^{-4}$ ×1.50**	Pen Dispt. mm	Deflec. in.$\times 10^{-4}$ ×0.80**	Pen Dispt. mm	Deflec. in.$\times 10^{-4}$ ×0.40**
0.	0	0	0	0		0	0	0
0.02	0	0	0	0	0	0	0	0
0.04	1.0	100.0	0.6	0.90	1.0	0.80	1.0	0.40
0.06	6.0	600.0	10.0	15.00	8.0	6.40	5.6	2.24
0.08	7.5	750.0	11.4	17.10	12.0	9.60	8.6	3.44
0.10	7.5	750.0	13.0	19.50	13.0	10.40	9.7	3.88
0.12	7.5	750.0	13.6	20.40	13.4	10.72	10.2	4.08
0.14	7.5	750.0	14.0	21.00	13.8	11.04	10.3	4.12
0.16	7.5	750.0	14.1	21.15	14.0	11.20	10.4	4.16
0.18	7.5	750.0	14.2	21.30	14.4	11.52	10.6	4.24
0.20	7.5	750.0	14.4	21.60	14.6	11.68	10.7	4.28
0.22	7.5	750.0	14.5	21.75	14.8	11.84	10.8	4.32
0.24	0	0	8.0	12.00	6.0	4.80	8.0	3.20
0.26	0	0	3.0	4.50	3.5	2.80	4.0	1.60
0.28	0	0	1.6	2.40	2.6	2.08	2.8	1.12
0.30	0	0	1.4	2.10	2.2	1.76	2.0	0.80
0.32	0	0	1.2	1.80	2.0	1.60	1.9	0.76
0.34	0	0	1.1	1.65	1.9	1.52	1.8	0.72
0.36	0	0	1.0	1.50	1.8	1.44	1.6	0.64
0.38	0	0	0.8	1.20	1.7	1.36	1.5	0.60
0.40	0	0	0.7	1.05	1.5	1.20	1.4	0.56
0.42	0	0	0.6	0.90	1.4	1.12	1.3	0.52
0.44	0	0	0.6	0.90	1.3	1.04	1.2	0.48
0.46	0	0	0.6	0.90	1.2	0.96	1.1	0.44
0.48	0	0	0.6	0.90	1.1	0.88	1.1	0.44
0.50	0	0	0.6	0.90	1.1	0.88	1.1	0.44

* The deflection functions recorded at distances 10.75 and 13.25 inches were too small to justify this type of data reduction.

** Calibration Factors, lbs./mm or in. $\times 10^{-4}$/mm

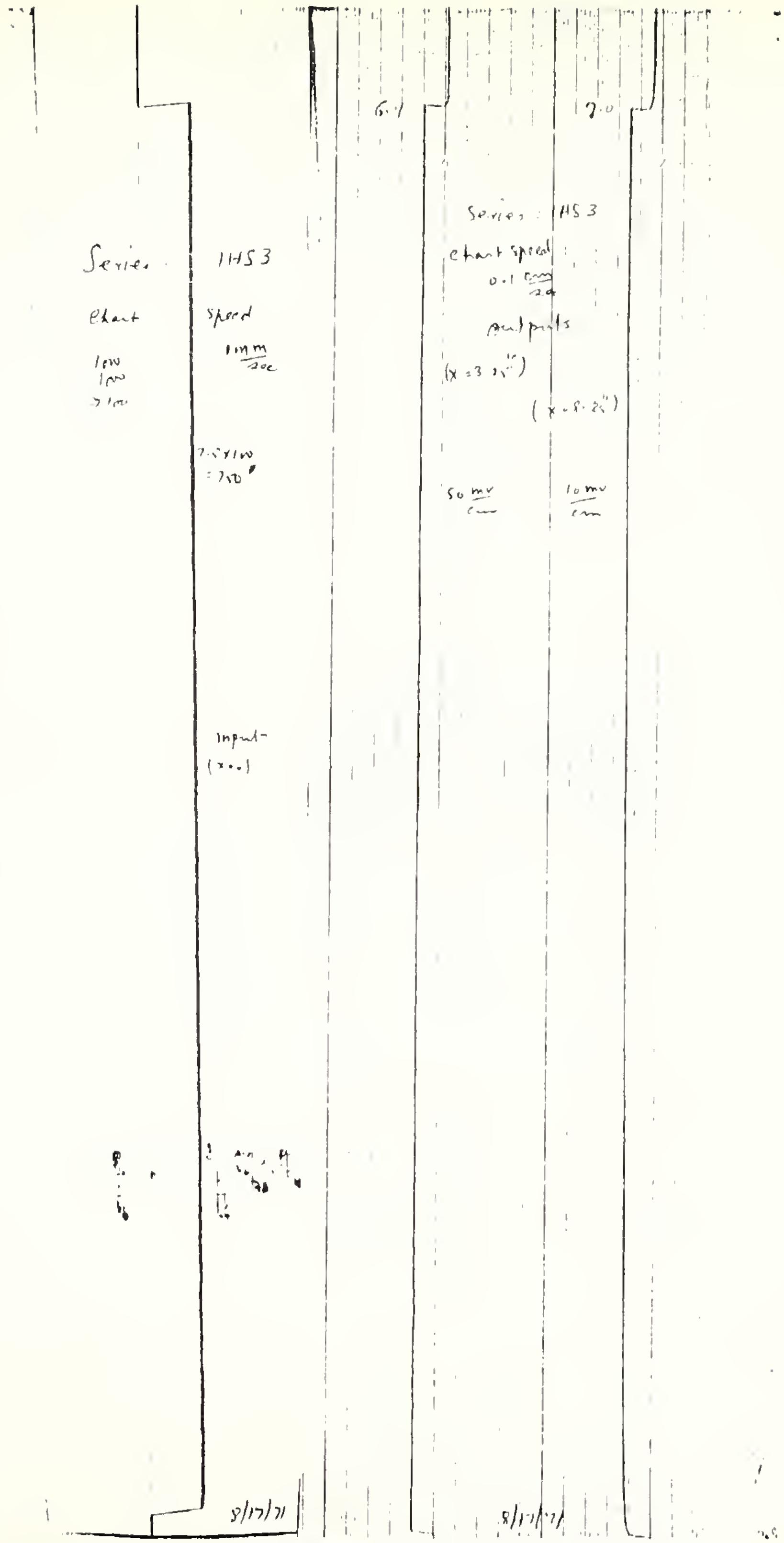

FIGURE 23 TYPICAL TRACES OF A STATIC LOAD TEST

the hydraulic actuator moved up and down from zero load to maximum and back to zero. This sequence was repeated automatically for the programmed number of 250 cycles at a rate of 15 cycles per minute.

Typical traces of repeated load input and deflection outputs are illustrated in Figure 24. Formats for data reduction are shown in Tables 6 and 7, respectively, for static and repeated load tests.

the hydraulic actuator moved up and down from zero load to maximum and back to zero. This sequence was repeated automatically for the programmed number of 250 [illegible] at a rate of 15 cycles per minute.

[illegible]

outputs [illegible]

[illegible]

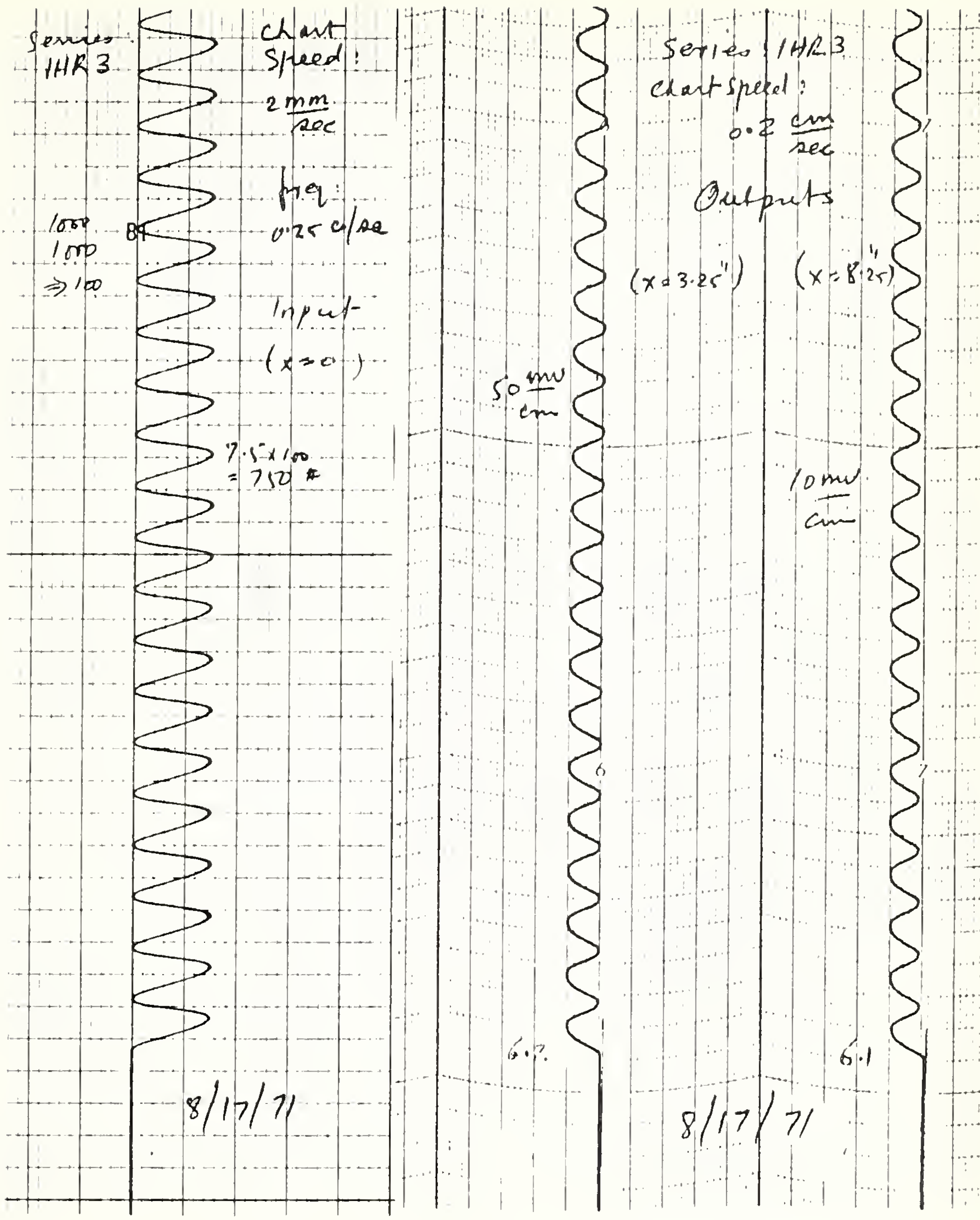

FIGURE 24 TYPICAL TRACES OF A REPEATED LOAD TEST

TABLE 6

TYPICAL DATA REDUCTION FOR STATIC TEST

Series: 1HS3

x in.	Preamp. Setting mv/cm	Calib. Factor in.$\times 10^{-4}$/mm	Pen Dispt. mm	Total Deflec. in.$\times 10^{-4}$
3.25	50	4.000	6.4	25.60
5.75	20	1.600	9.0	14.40
8.25	10	0.800	7.0	5.60
10.75	5	0.375	2.0	0.75
13.25	2	0.140	0.0	0.0

TABLE 7

TYPICAL DATA REDUCTION FOR REPEATED TEST

Series: 1HR3

x in.	Preamp. Setting mv/cm	Calib. Factor in.$\times 10^{-4}$/mm	Pen Dispt. mm	Total Deflec. in.$\times 10^{-4}$
3.25	50	4.000	6.2	24.80
5.75	20	1.600	8.2	13.12
8.25	10	0.800	6.1	4.88
10.75	5	0.375	1.9	0.71
13.25	2	0.140	0.0	0.0

RESULTS AND DISCUSSION

The theory and experimental techniques to evaluate the response behavior of a pavement system were advanced in previous sections. The test data and analysis of results are presented in this Section and Appendix A together with their implications and the indicated relationships. The scope of the investigation included two flexible model pavements, differing in the thickness of the surface course, tested at three stress levels and three temperatures. Impulse, static and repeated loads were applied at the center of the pavements. Corner loadings were also conducted to investigate the edge effects that might result as a consequence of the sides of the box. For all series, surface deflections were recorded as functions of time at five locations. The results are presented and discussed in the following sequence:

1. Impulse Load Test Results
    a. Normal Distribution and the Deflection Basin
    b. Response Function of Flexible Pavements
    c. Form of the Response Function
    d. Check for the Response Function
    e. Load-Independency of the Response Function
    f. Effect of Temperature on the Response Function

g. Effects of Surface Course Thickness and Spatial Location

2. Static Load Results

3. Repeated Load Results

4. Corner Loading Test Results

Impulse Load Test Results

The peak load magnitudes employed in most of the impulse tests were 187.5, 375.0 and 750.0 lbs. These correspond, respectively, to applied pressures of 14.93, 29.86 and 59.72 psi. The time duration was 0.16 second. This was established by adjusting the drop height of the hydraulic actuator and the frequency of the MTS function generator. The capability of the testing equipment was a limiting factor in using time duration less than 0.16 second.

Normal Distribution and the Deflection Basin

The peak values from the deflection functions are presented in Table 8 and plotted in Figures 25 through 30. These data indicate that in all cases the deflections increase with increase in temperature. This was to be expected since asphaltic materials are thermoplastic and an increase in temperature will result in increase of their flow characteristics.

As the thickness of a pavement structure is increased, the resistance to imposed loads increases and the system tends to deform less. However, this behavior was not uniform

TABLE 8

PEAK DEFLECTIONS FROM IMPULSE TESTS

Series	$y(x)$, inch ($\times 10^{-4}$) Distance from Load Center, inches 3.25	5.25	8.25	10.75	13.25
1LI1	1.50	0.64	0.16	--	--
1LI2	2.48	0.77	0.32	--	--
1LI3	7.65	3.04	0.67	--	--
1MI1	3.23	1.52	0.38	--	--
1MI2	7.50	4.00	0.86	--	--
1MI3	15.00	8.00	2.00	--	--
1HI1	4.05	2.24	0.78	0.03	--
1HI1*	4.28	2.24	0.86	0.08	--
1HI2	11.70	6.16	2.11	0.15	--
1HI3	19.20	10.40	3.44	0.36	--
1HI3*	21.75	11.84	4.32	0.57	--
2LI1	0.83	0.48	0.16	--	--
2LI1*	0.75	0.42	0.18	--	--
2LI2	1.43	0.83	0.22	--	--
2LI2*	1.43	0.86	0.29	--	--
2LI3	4.88	3.36	1.36	0.69	0.20
2MI1	1.91	1.40	0.74	0.38	0.22
2MI2	3.75	2.76	1.44	0.75	0.32
2MI2*	3.64	2.52	1.09	0.68	0.25
2MI3	8.10	5.04	2.48	1.28	0.49
2MI3*	8.40	5.12	2.52	1.43	0.53
2HI1	3.60	2.40	1.04	0.68	0.22
2HI1*	3.30	2.40	1.12	0.68	0.25
2HI2	7.20	5.08	2.64	1.43	0.78
2HI2**	6.90	4.64	2.16	1.20	0.48
2HI2**	6.90	4.80	2.24	1.28	0.48
2HI3	12.00	9.92	4.48	2.70	1.05

* Represent duplicate series of tests.

** Represent triplicate series of tests.

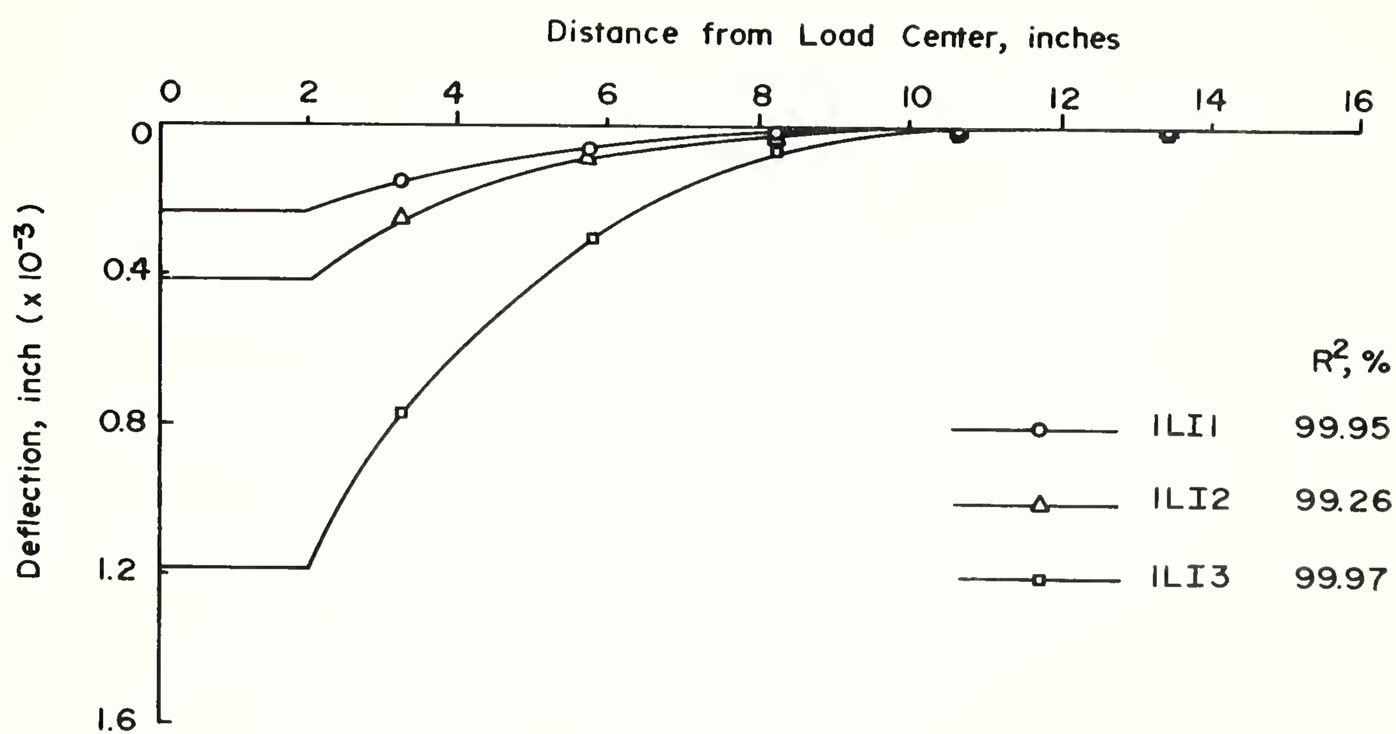

FIGURE 25 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS — 1- INCH SURFACE, 50 °F

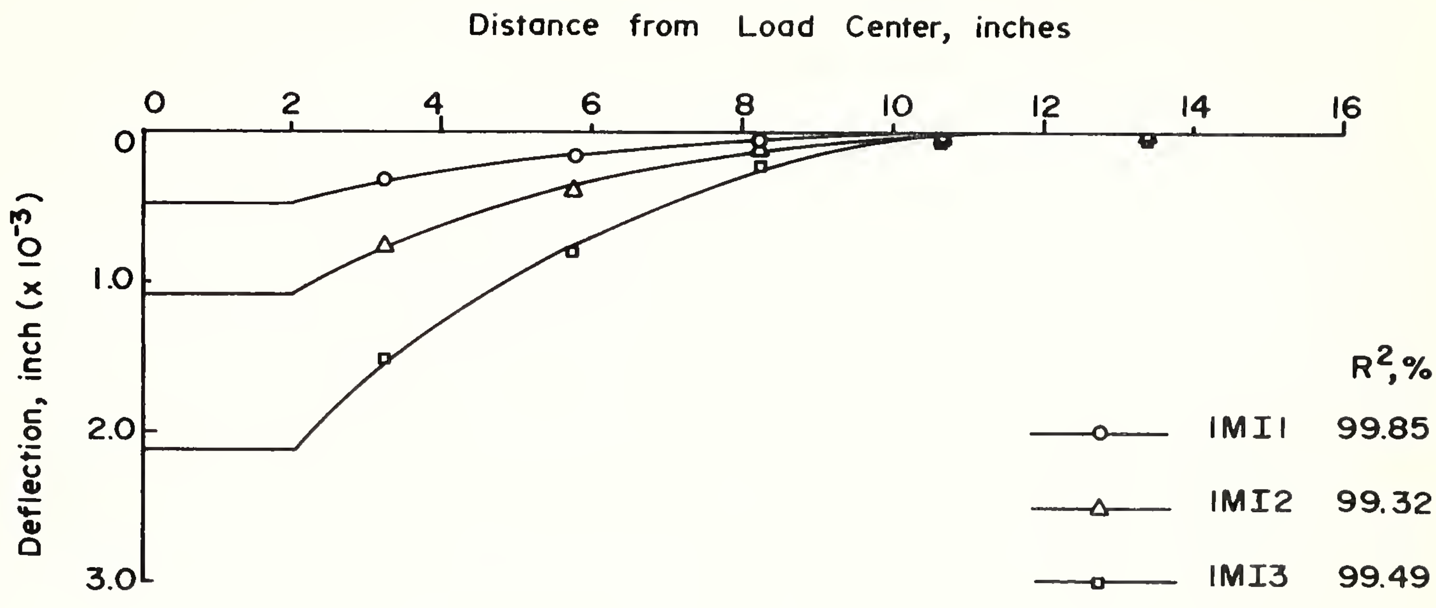

FIGURE 26 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS— 1-INCH SURFACE, 75°F

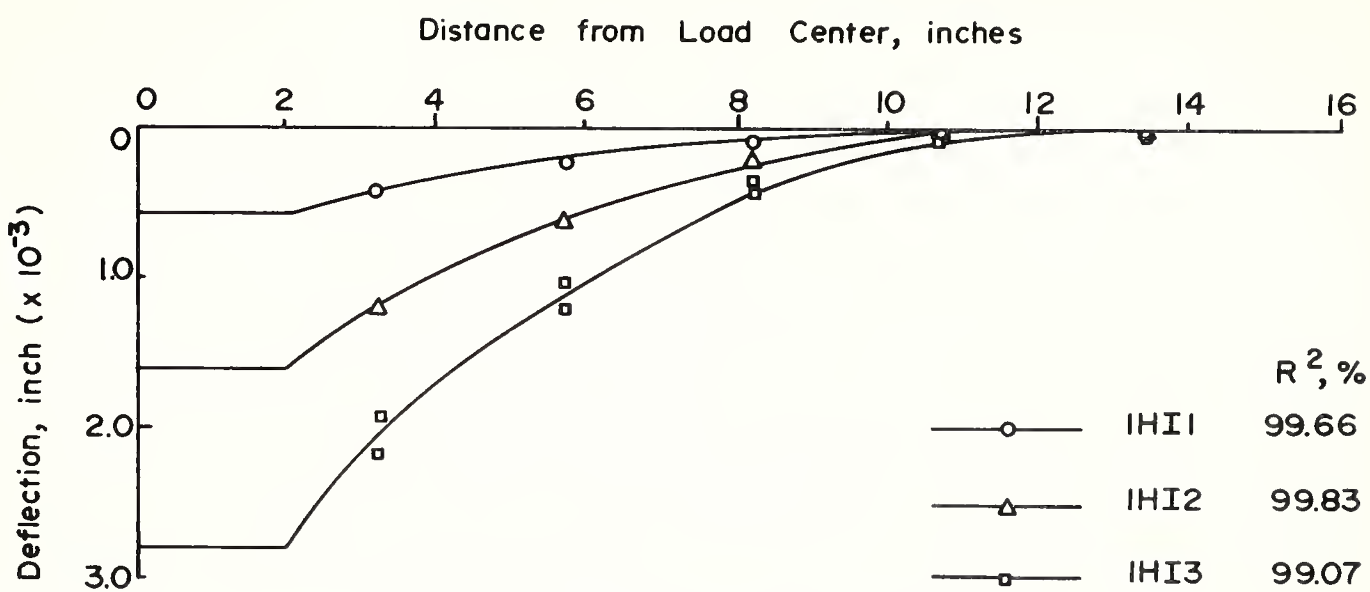

FIGURE 27 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS— 1-INCH SURFACE, 100 °F

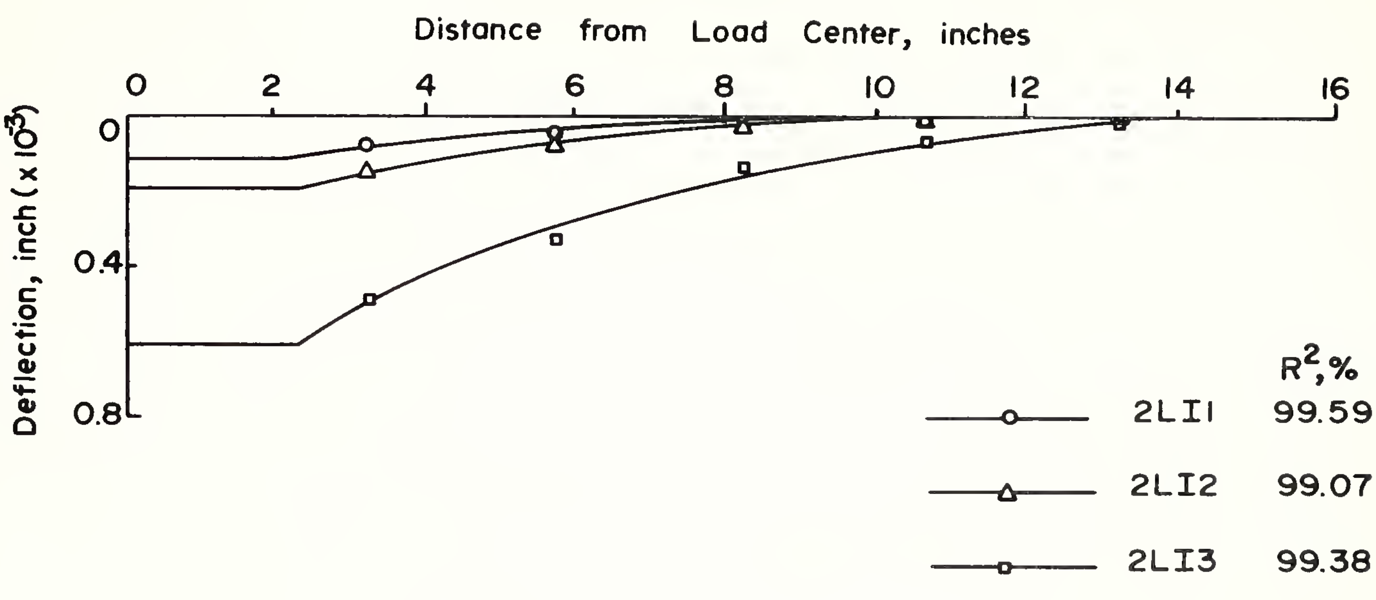

FIGURE 28 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS— 2-INCH SURFACE, 50°F

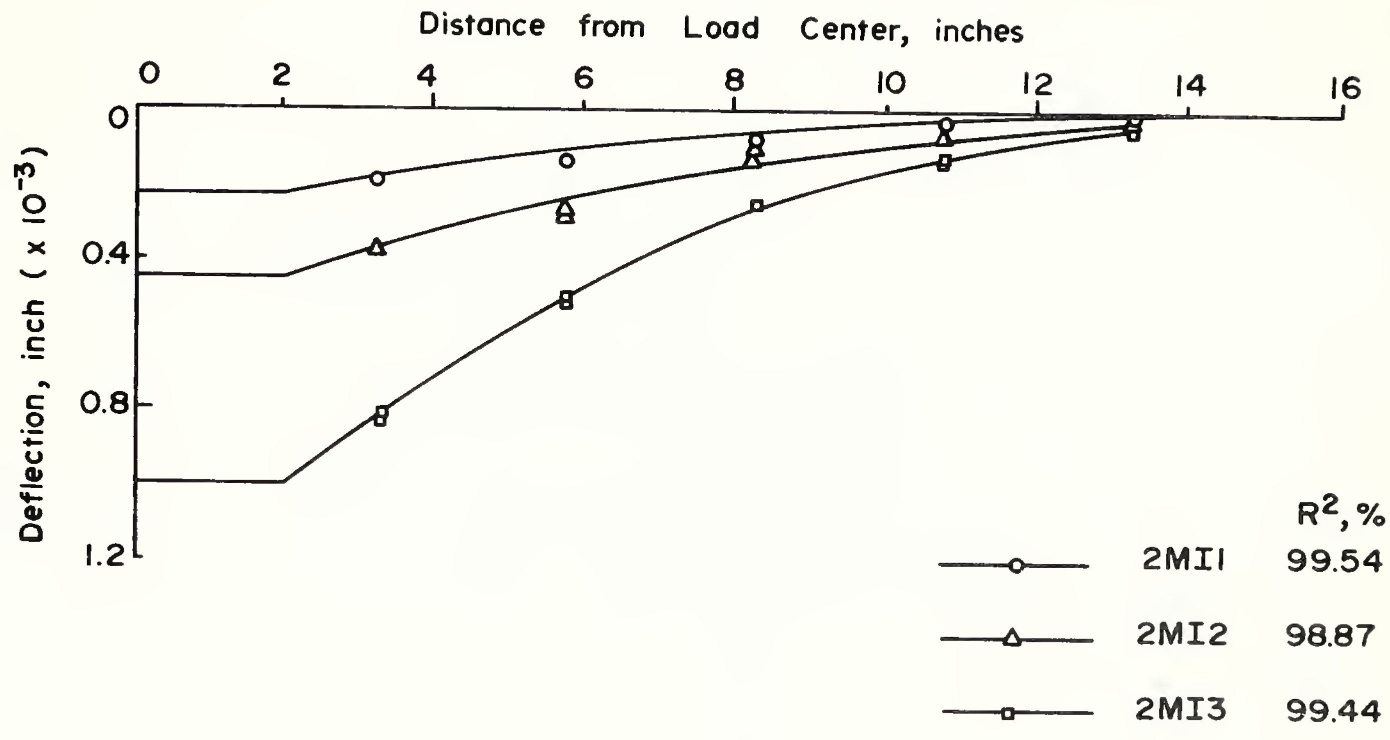

FIGURE 29 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS — 2-INCH SURFACE, 75°F

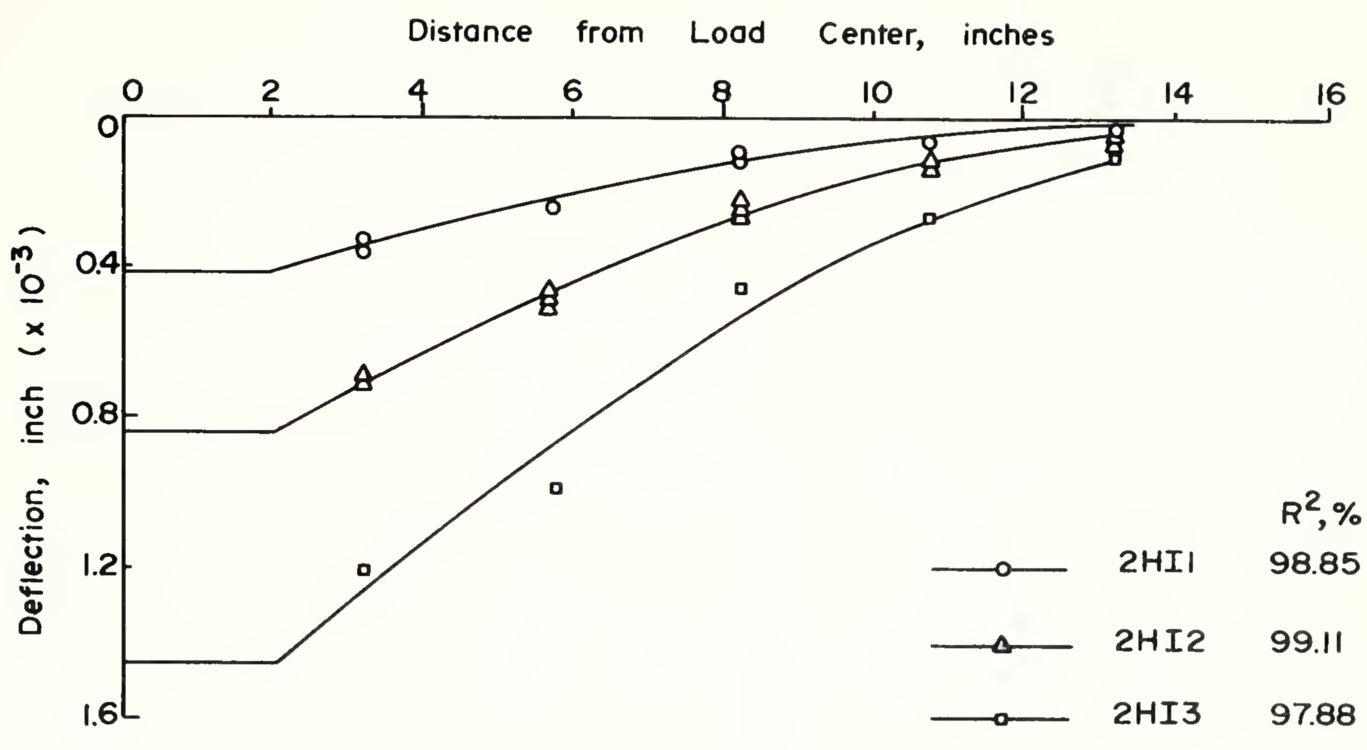

FIGURE 30 PEAK DEFLECTION PROFILES FROM IMPULSE TESTS— 2-INCH SURFACE, 100°F

regarding the spatial locations x where measurements were made. At the two locations closest to the loaded position, decreased deflections were observed with increase in thickness, while at the other three locations, namely 8.25 inches and beyond, deflections increased with increasing thickness. See the typical comparisons made in Figure 31. This phenomenon was attributed to the slab action of the pavement system. When a system with a thicker surface is subjected to load, it will demonstrate less flexibility than would be the case for a thinner pavement. Consequently, the rate of decrease in deflection with increasing x is less than that for a thinner surface.

The function selected to fit the peak deflection data of Table 8 was of the form

$$y(x) = y_0 e^{-Dx^2} \qquad (32)$$

where $y(x)$ = the deflection at a distance x from the load center,

y_0 = the maximum deflection of the deflected basin, and

D = a constant, reflecting the attenuation of the deflected basin with x.

The values of y_0 and D determined by a non-linear regression method are given in Table 9. The non-linear regression analysis gave squared correlation coefficients (R^2), as reported in Table 9. These range from a little less than

regarding the spatial locations where measurements were made. At the two locations closest to the model position, decreased deflections were [illegible] ... heads, while at the [illegible] ... the bend[illegible] [illegible]

[illegible]

The value[illegible] [illegible] method are given in Table [illegible] analysis gave squared [illegible] reported in Table 9. These [illegible]

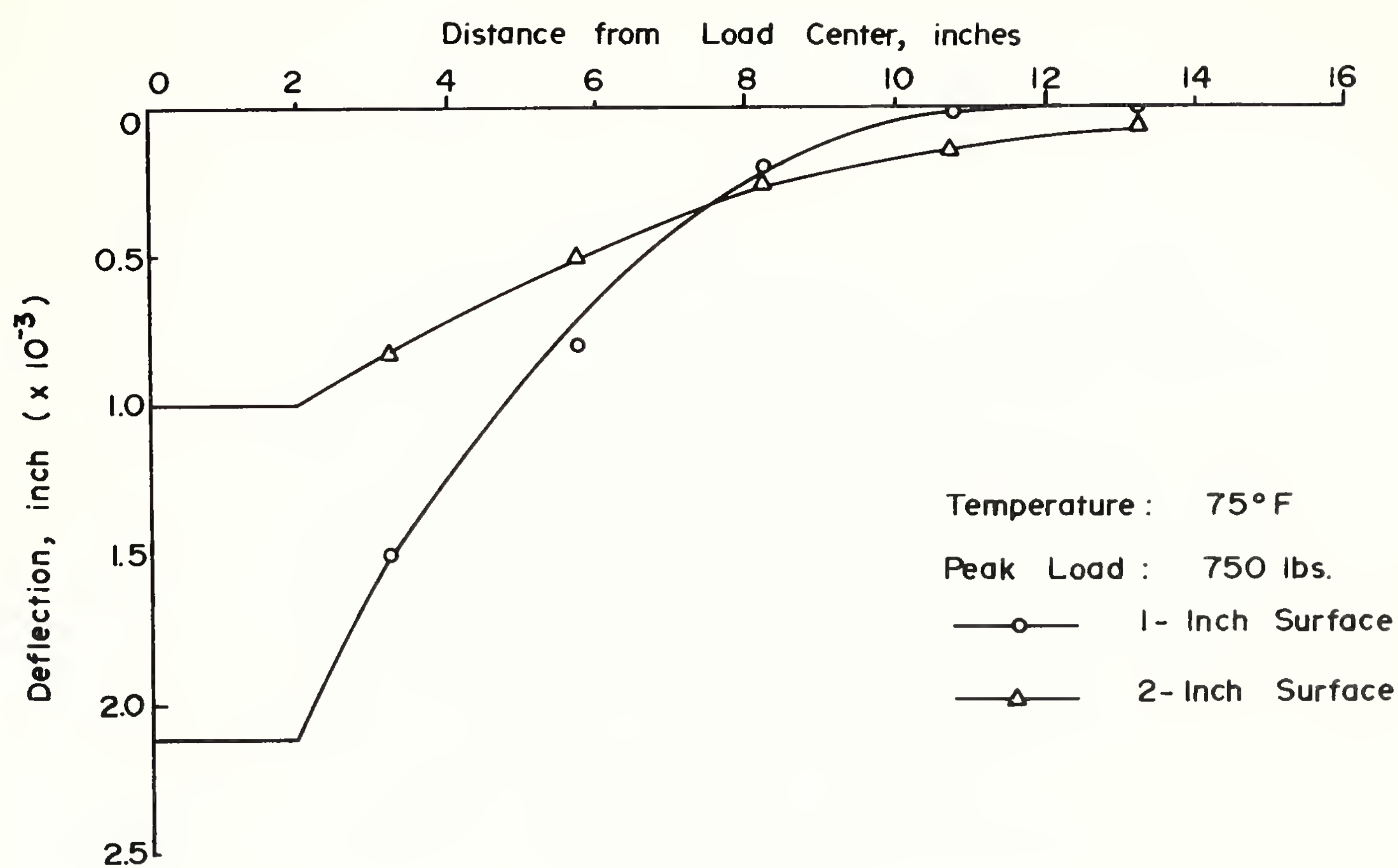

FIGURE 31 TYPICAL COMPARISON OF DEFLECTION PROFILES FROM IMPULSE TESTS

TABLE 9

VALUES OF y_0 AND D IN THE EQUATION $y(x) = y_0 e^{-Dx^2}$

Series	y_0 in. ($\times 10^{-4}$)	D 1/in.^{2}	R^2 %	Number of Points
1LI1	2.256	0.0385	99.95	5
1LI2	4.071	0.0475	99.26	5
1LI3	11.881	0.0416	99.97	5
Average		0.0425		
1MI1	4.716	0.0353	99.85	5
1MI2	10.723	0.0325	99.32	5
1MI3	21.236	0.0317	99.49	5
Average		0.0332		
1HI1	5.676	0.0288	99.66	10
1HI2	16.124	0.0299	99.83	5
1HI3	27.907	0.0287	99.07	10
Average		0.0291		
2LI1	1.066	0.0274	98.88	10
2LI2	1.959	0.0281	99.07	10
2LI3	6.106	0.0199	99.38	5
Average		0.0251		
2MI1	2.254	0.0153	99.54	5
2MI2	4.437	0.0168	98.87	10
2MI3	9.997	0.0195	99.44	10
Average		0.0172		
2HI1	4.163	0.0176	98.85	10
2HI2	8.388	0.0172	99.11	15
2HI3	14.560	0.0149	97.88	5
Average		0.0166		

0.98 to close to unity. This is an indication that the model chosen to fit the data of deflected profiles is adequate to the task.

It is observed from Figures 25-31 that the portion of the pavement surface under the loading plate was assumed to have experienced a constant displacement at the indicated test conditions. This was a consequence of the use of a rigid loading plate. The choice of the rigid plate itself was made to obtain a distinct trace of load duration during impulse tests.

The effect of temperature on y_0 is shown in Figures 32 and 33, respectively, for the model pavements having one-inch and two-inch surface courses. As temperature increases, the less stiff and more responsive the material becomes. Thus, for both model pavements, y_0 increases with increases in temperature. However, the plots of y_0 versus temperature do exhibit differences between the two cases. For the one-inch surface, the bottom layers, whose response constitutes most of the total deflection, are less affected by temperature changes than the bituminous surface course and hence the rate of increase in y_0 decreases with increasing temperature. On the other hand, as the thickness of the asphaltic concrete surface is increased, the influence of the surface layer becomes more pronounced. Inasmuch as the bituminous mixture is more temperature susceptible than the other layer materials, increasing temperature increases the rate of

0.95 to close to unity. This is an indication that the model chosen to fit the data of deflected gravity is adequate to the task.

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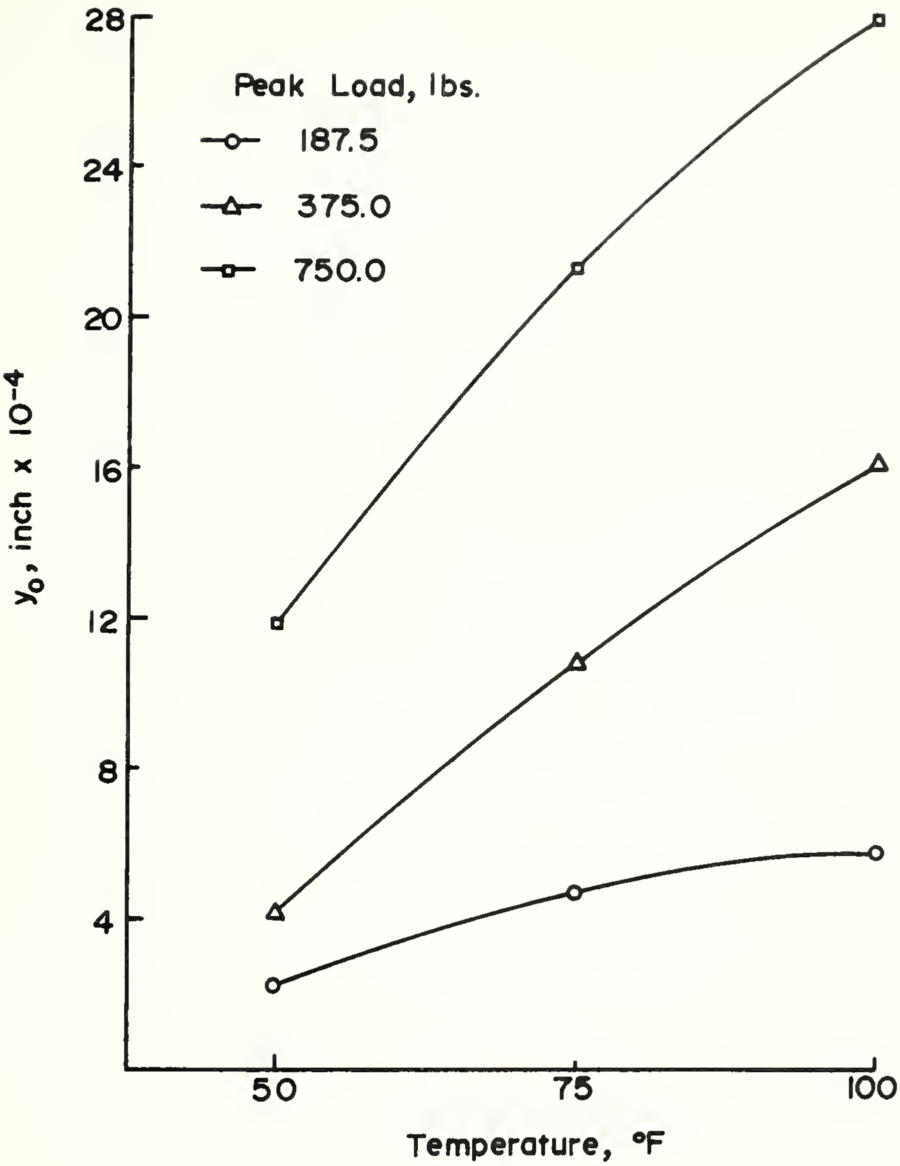

FIGURE 32 y_o VERSUS TEMPERATURE — 1- INCH SURFACE

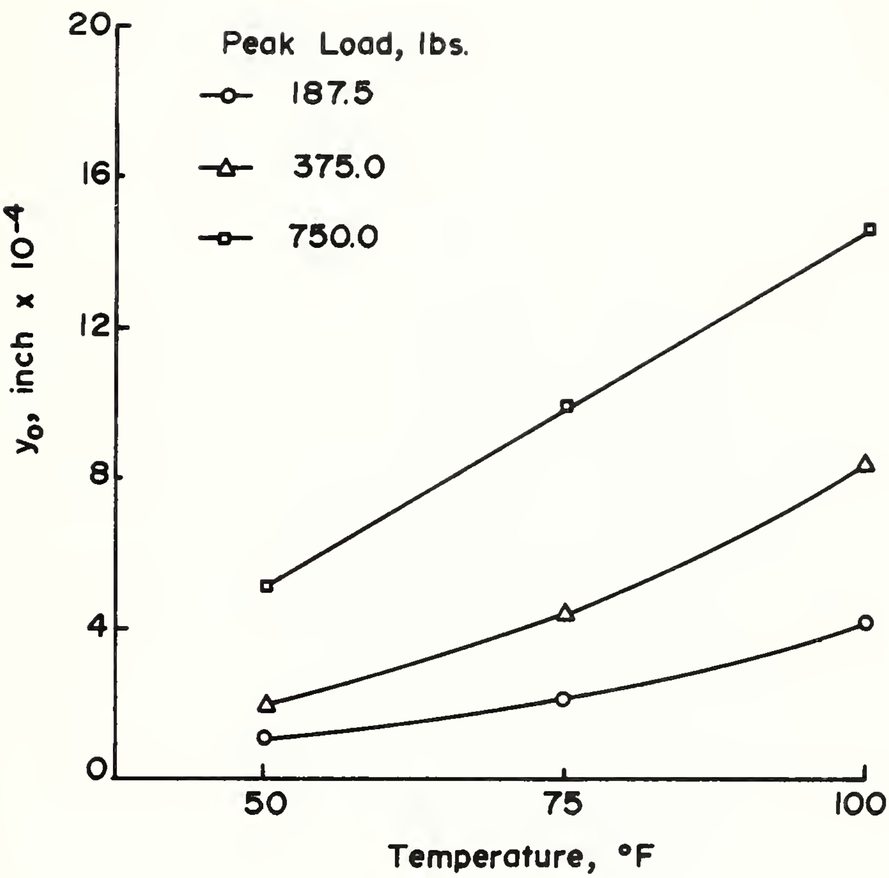

FIGURE 33 y_o VERSUS TEMPERATURE — 2-INCH SURFACE

increase in y_0. This is also considered as evidence in this investigation that the subgrade and the base course, in addition to the surface layer, responded to the input loads.

Figure 34 depicts the effect of surface course thickness on y_0 for the three load magnitudes and the three test temperatures. As the temperature increases the slopes of the lines increase, supporting the above discussion about the effect of increasing surface course thickness on y_0.

The factor D in Equation 32 is a measure of how the deflected basin attenuates with spatial distance x. Smaller D values reflect more spreading of the deflection basin. Table 9 indicates that the D parameter does not change appreciably, even though the magnitude of the peak load is increased at any one thickness or temperature. Average values of D versus temperature are plotted in Figure 35 for the two thicknesses. Inasmuch as increases in temperature increase pavement response for a given thickness, attenuation of the deflection basin with x decreases and hence D decreases. The figure also indicates that the slab action is reflected in the parameter D. As the surface course thickness is increased, the deflected basin spreads farther and thus D decreases. It appears from Figure 35 that there is no interaction between the effects of temperature and surface course thickness on D.

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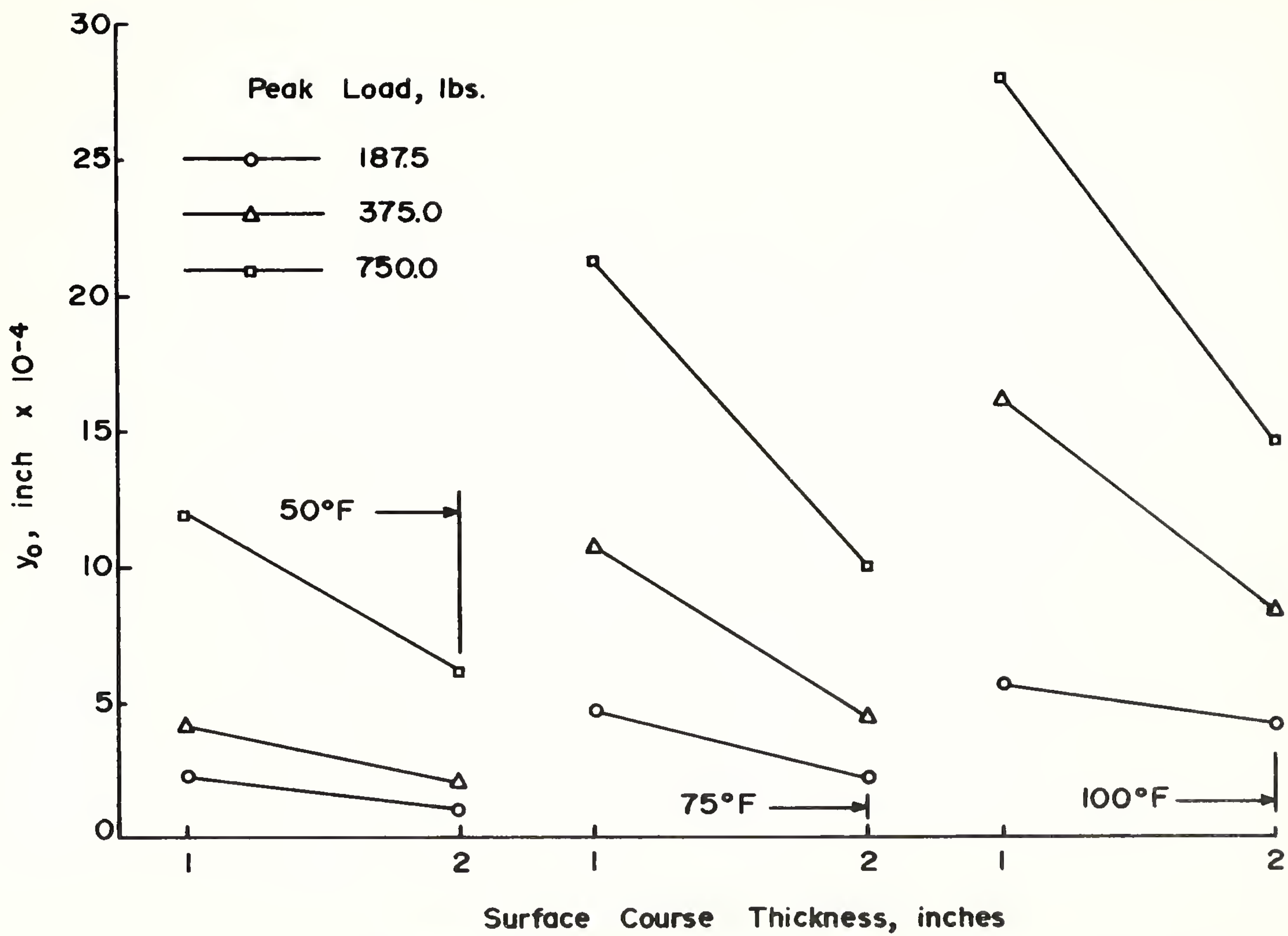

FIGURE 34 y_o VERSUS SURFACE THICKNESS — 50, 75, 100 °F

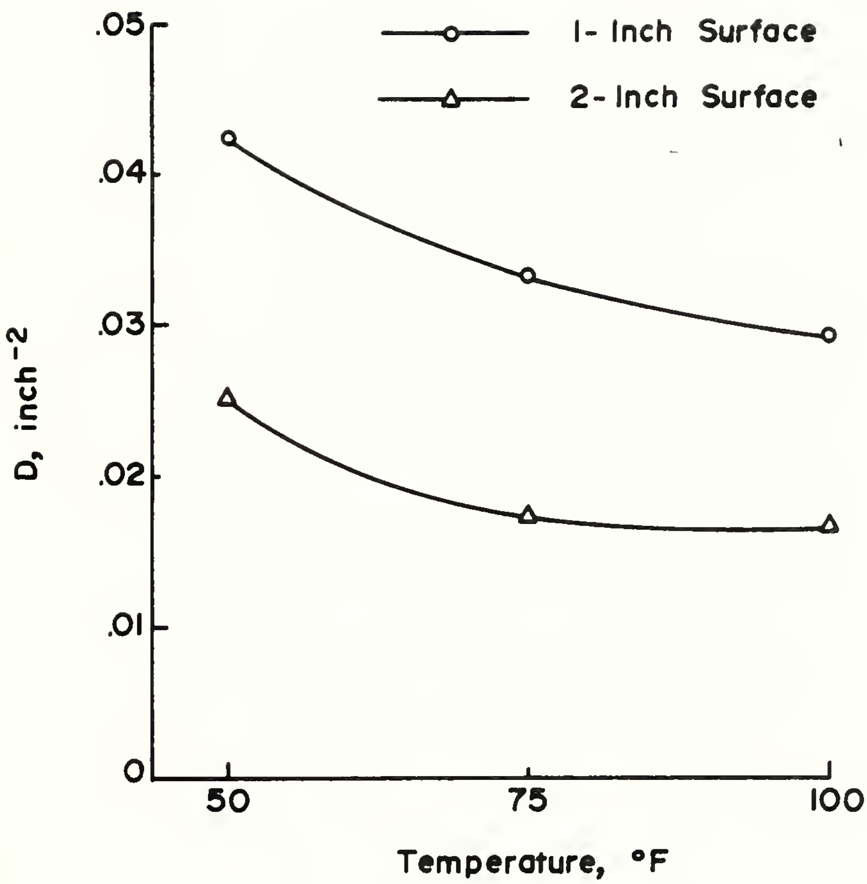

FIGURE 35 D VERSUS TEMPERATURE — 1- INCH, 2- INCH SURFACES

Response Function of Flexible Pavements

The response functions were calculated from the input load and the output deflection functions using Equation 13. Computations were made by the computer utilizing the subroutine CONVLI of the program listed in Appendix A. The results of the calculations are also shown in Appendix A. For each test series, the table of the input and outputs is followed by the response function table. The three parameters are given as functions of time. The third table in each series is the deflection function predicted by Equation 15 and as computed by subroutine CONVLE.

Typical plots of the input and outputs are shown in Figures 36, 37 and 38, and the corresponding response functions are illustrated in Figures 39, 40 and 41. It is seen that the response functions reach a first peak and then oscillate with time. The peak, in general, occurs in the time band of 0.04 to 0.08 seconds. The general shape of the response functions and the corresponding results agree with those obtained from mechanical models [45, 49, 50]. However, when mechanical models are employed to describe pavement behavior, the parameters must be defined initially, usually in the form of differential equations. Furthermore, no reliable method exists today to determine these parameters "ab initio." This study is characterized by the fact that the response function of a pavement system is determined directly from the input-output data of impulse tests. The

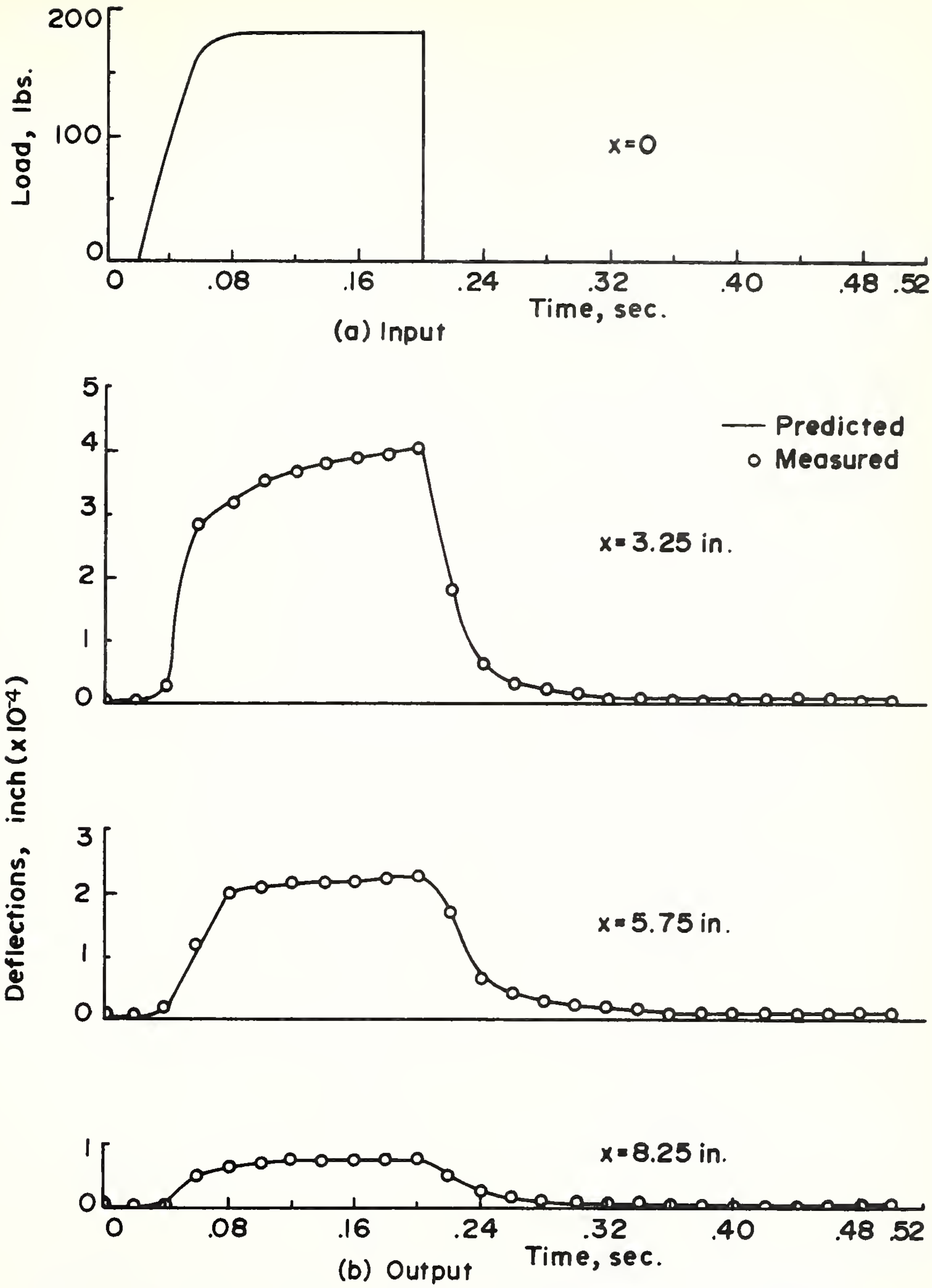

FIGURE 36 INPUT LOAD AND OUTPUT DEFLECTION FUNCTIONS — DATA FOR IHII

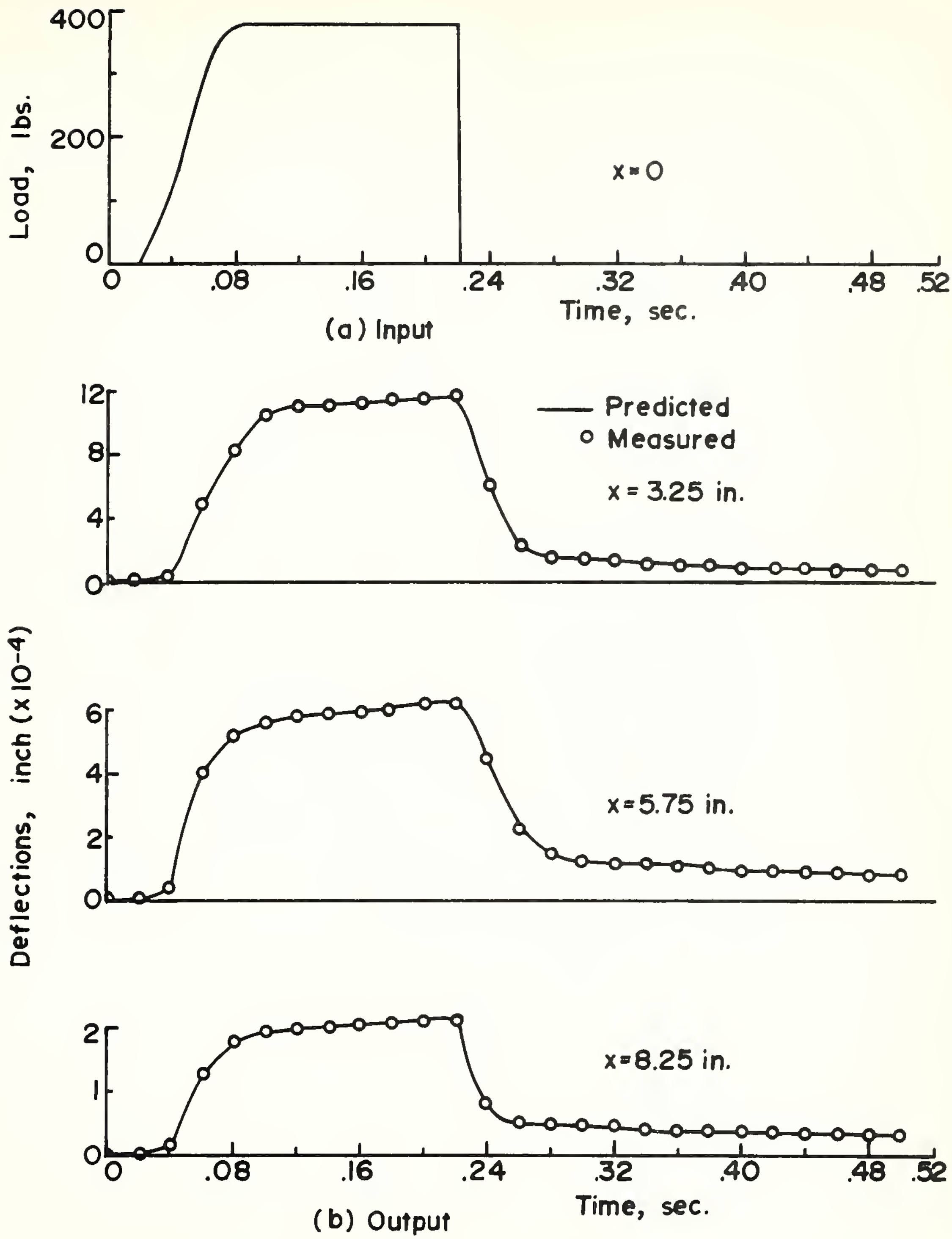

FIGURE 37 INPUT LOAD AND OUTPUT DEFLECTION FUNCTIONS—DATA FOR 1HI2

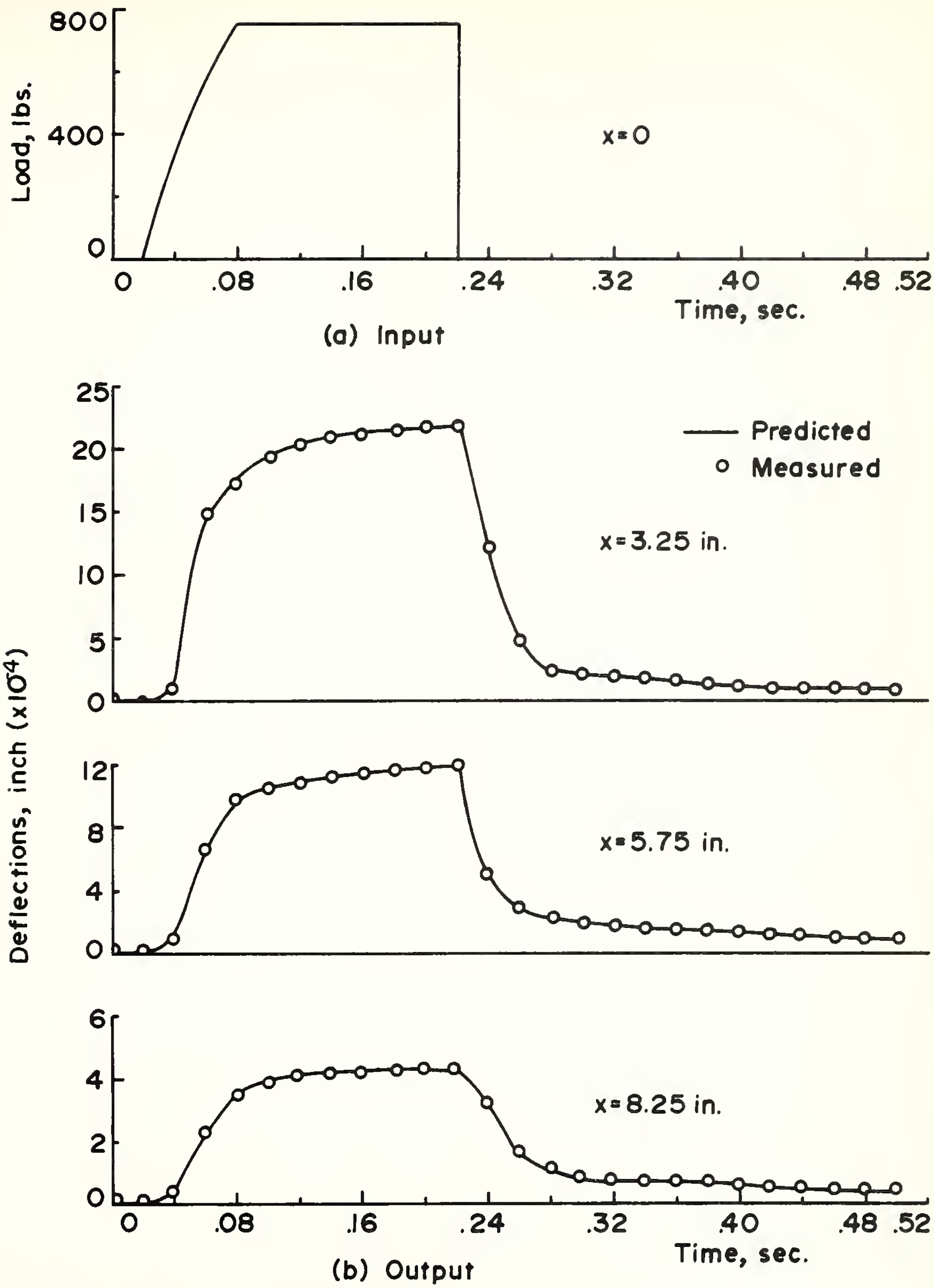

FIGURE 38 INPUT LOAD AND OUTPUT DEFLECTION FUNCTIONS — DATA FOR 1HI3

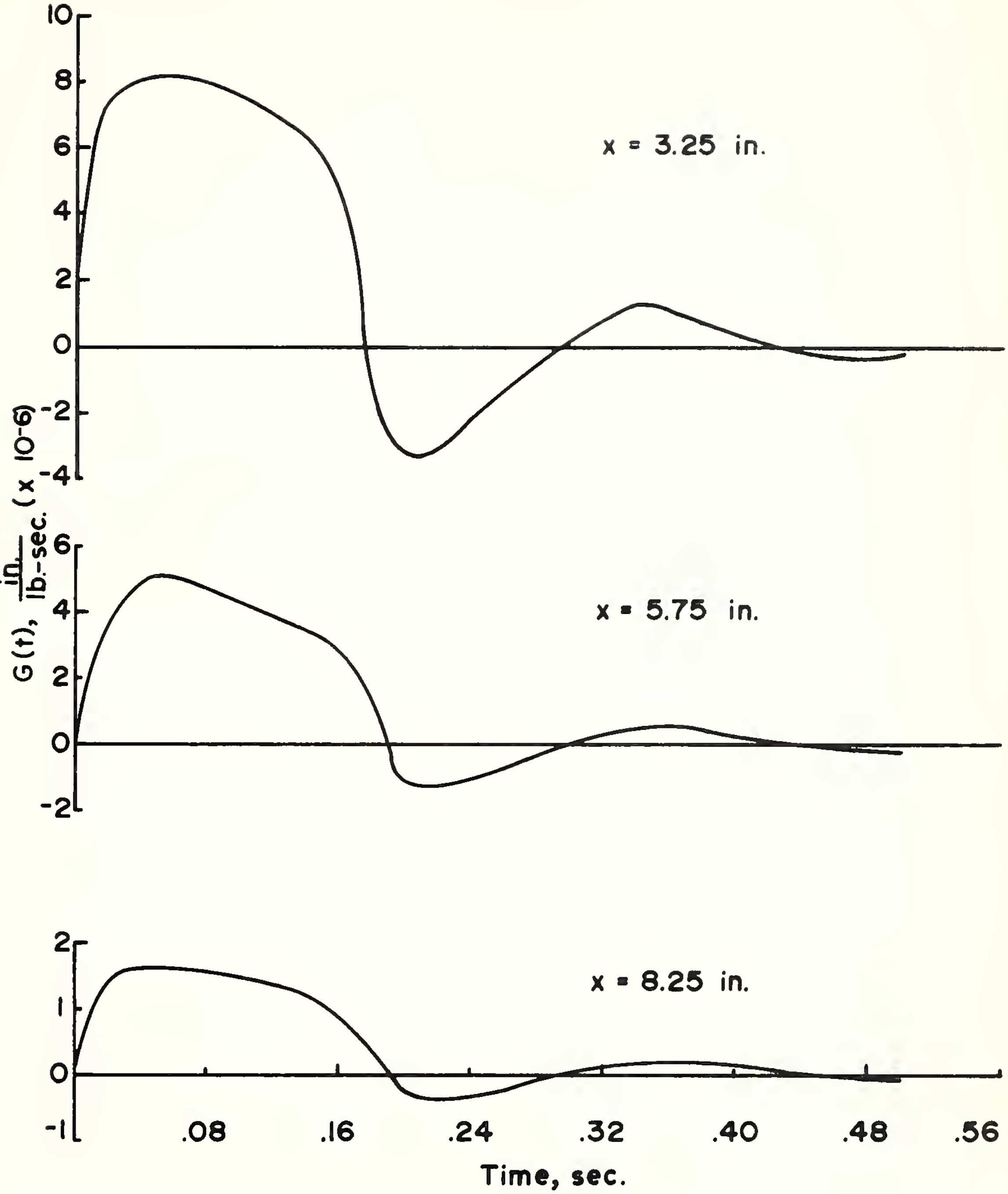

FIGURE 39 RESPONSE FUNCTIONS — DATA FROM IHII

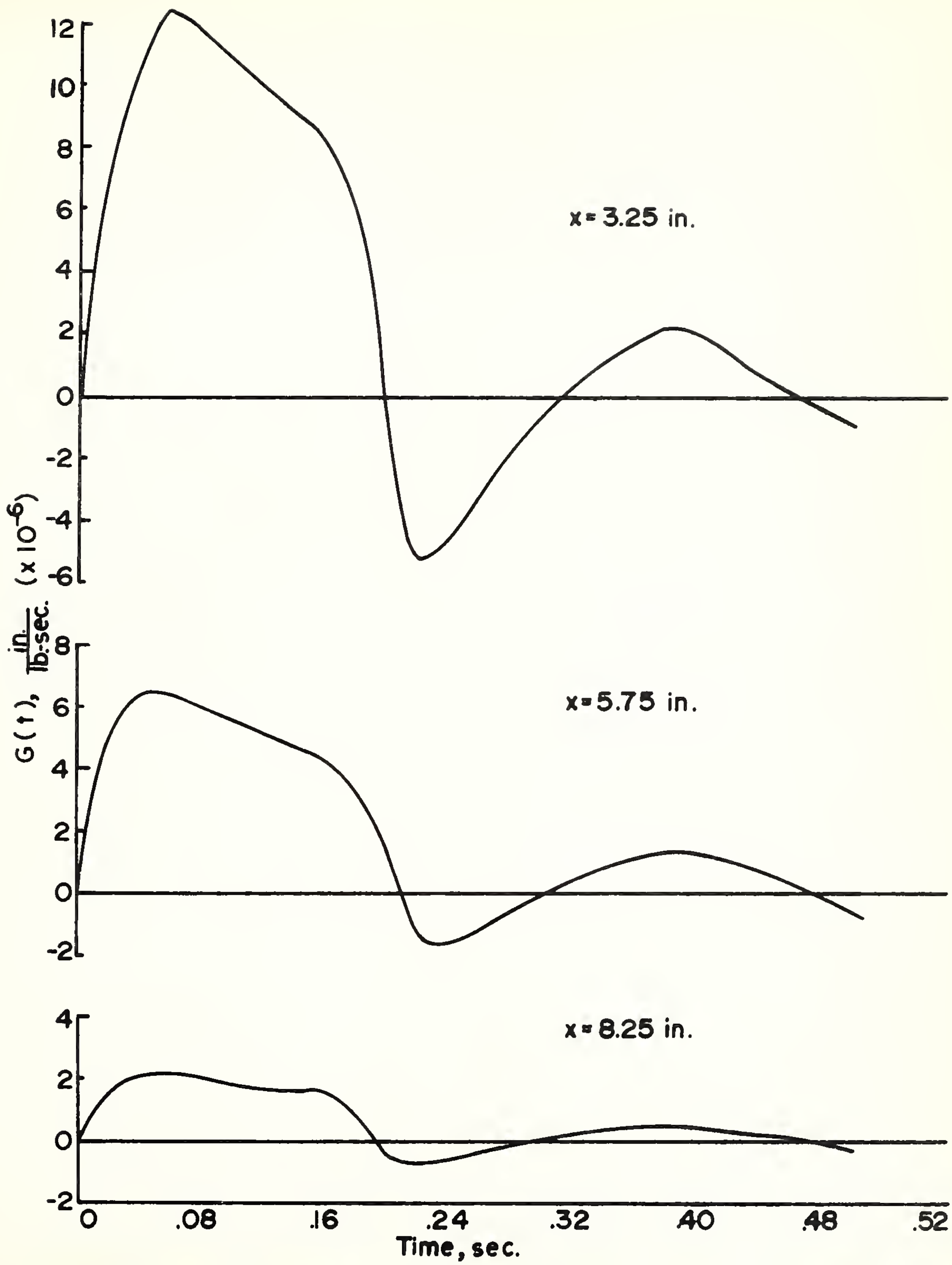

FIGURE 40 RESPONSE FUNCTIONS — DATA FROM IHI2

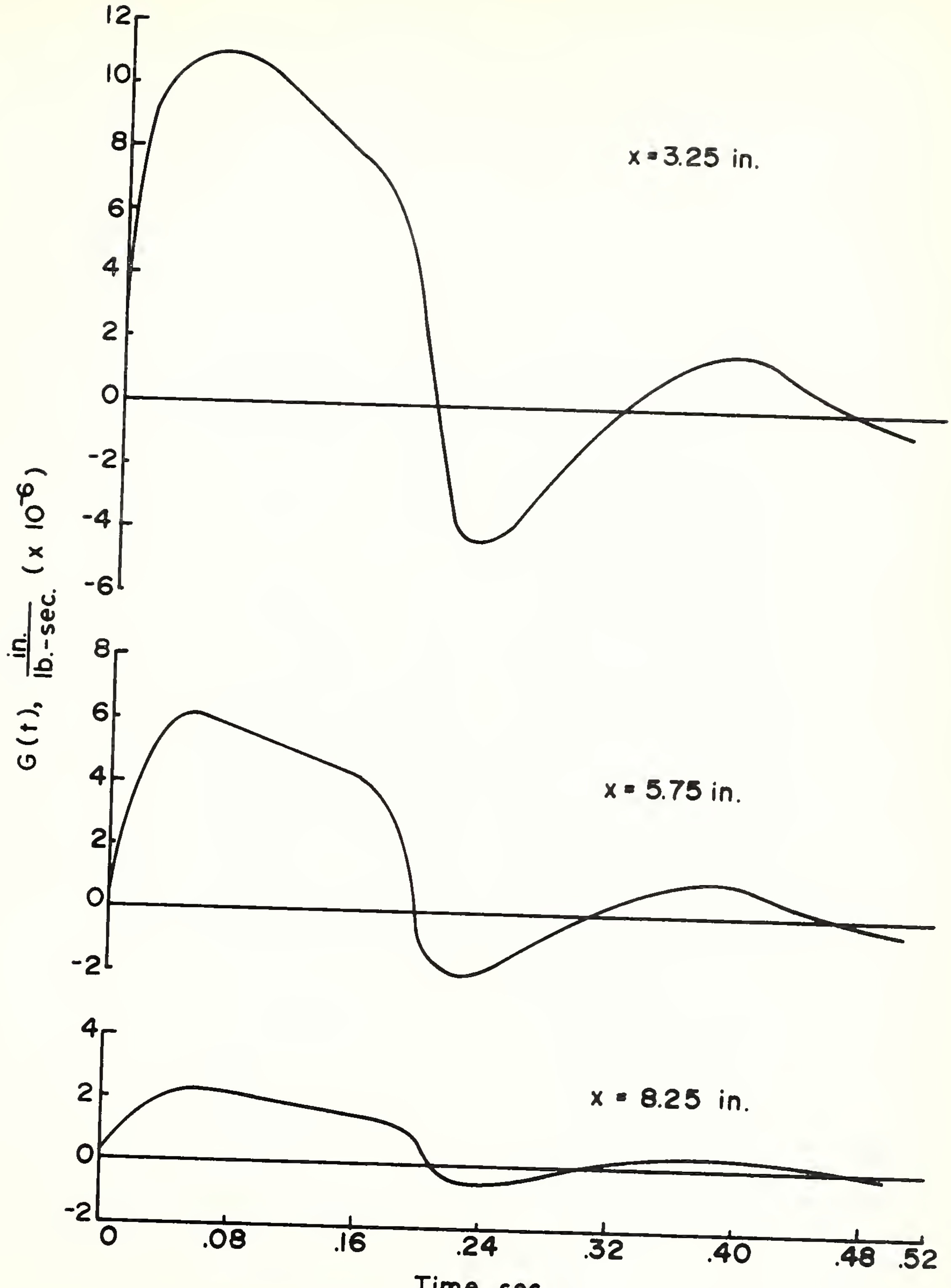

FIGURE 41 RESPONSE FUNCTIONS—DATA FROM 1H13

consistency of the response functions obtained in this study appears to validate the first assumption that "a determinable relationship exists between known input and output of a pavement system."

Form of the Response Function

Some typical response functions are represented in Figures 39, 40 and 41 for the indicated test conditions. In the interest of obtaining more information about pavement response functions and to provide a rational interpretation of the results, the response function curves were approximated by the mathematical model

$$G(t) = \alpha e^{-\beta t} \sin \gamma t \qquad (14)$$

where α, β and γ are parameters to be determined by a non-linear regression analysis. The results of the analysis are shown in Tables 10, 11 and 12. It is seen from these tables that the squared correlation coefficient (R^2) ranged from 66 to 93 percent.

From the technique followed in approximating the response function curve, it is obvious that the number of parameters required to describe the response function depends on the model selected. As the number of parameters in the model increases, the accuracy of curve-fitting improves and, consequently, the value of R^2 increases.

TABLE 10

DESCRIPTIVE PARAMETERS IN THE RESPONSE FUNCTIONS AT 50°F

$$G(t) = \alpha e^{-\beta t} \sin \gamma t$$

Thickness in.	x in.	F_p lbs.	α $\frac{\text{in.}}{\text{lb.-sec.}}$ $\times 10^{-6}$	β $\frac{1}{\text{sec.}}$	γ $\frac{1}{\text{sec.}}$	R^2 %
1	3.25	375.0	6.86	10.764	13.382	85.73
		750.0	10.40	10.551	15.848	87.48
		Average	8.63	10.658	14.615	
	5.75	375.0	1.36	7.127	14.593	78.09
		750.0	3.60	8.784	17.285	86.79
		Average	2.48	7.956	15.939	
	8.25	375.0	0.48	6.140	14.470	75.86
		750.0	0.72	8.195	15.716	86.32
		Average	0.60	7.168	15.093	
2	3.25	187.5	5.39	12.879	12.336	92.39
		375.0	3.57	9.847	12.709	87.60
		750.0	5.39	8.785	16.018	84.34
		Average	4.78	10.504	13.688	
	5.75	187.5	2.41	11.302	14.222	83.88
		375.0	1.70	7.517	12.988	83.16
		750.0	3.51	7.540	16.406	84.82
		Average	2.54	8.786	14.539	
	8.25	187.5	0.70	9.610	12.598	65.68
		375.0	0.51	7.409	12.996	73.44
		750.0	1.56	8.792	16.824	83.86
		Average	0.92	8.604	14.139	

TABLE 11

DESCRIPTIVE PARAMETERS IN THE RESPONSE FUNCTIONS AT 75°F

$$G(t) = \alpha e^{-\beta t} \sin \gamma t$$

Thick-ness in.	x in.	F_p lbs.	α $\frac{\text{in.}}{\text{lb.-sec.}}$ $\times 10^{-6}$	β $\frac{1}{\text{sec.}}$	γ $\frac{1}{\text{sec.}}$	R^2 %
1	3.25	187.5	21.28	12.604	13.875	88.26
		375.0	21.89	11.537	14.989	86.00
		750.0	24.61	12.348	13.752	87.44
		Average	22.59	12.163	14.205	
	5.75	187.5	10.31	13.079	12.791	85.77
		375.0	13.73	12.908	13.496	88.41
		750.0	14.42	13.084	12.478	89.07
		Average	12.82	13.024	12.922	
	8.25	187.5	2.08	10.434	11.703	87.44
		375.0	2.52	11.177	15.026	87.98
		750.0	3.51	13.199	13.934	87.35
		Average	2.70	11.603	13.554	
2	3.25	187.5	9.49	9.500	13.868	90.80
		375.0	7.98	8.232	12.914	88.23
		750.0	12.26	11.000	15.272	88.52
		Average	9.91	9.577	14.351	
	5.75	187.5	6.97	9.525	14.191	90.31
		375.0	4.97	7.164	14.398	81.43
		750.0	9.17	13.347	14.020	89.63
		Average	7.04	10.012	14.203	
	8.25	187.5	4.21	10.523	13.452	92.32
		375.0	3.53	9.195	14.479	91.03
		750.0	3.73	11.471	15.260	87.63
		Average	3.82	10.396	14.397	

TABLE 12

DESCRIPTIVE PARAMETERS IN THE RESPONSE FUNCTIONS AT 100°F

$$G(t) = \alpha e^{-\beta t} \sin \gamma t$$

Thickness in.	x in.	F_p lbs.	α $\frac{\text{in.}}{\text{lb.-sec.}}$ $\times 10^{-6}$	β $\frac{1}{\text{sec.}}$	γ $\frac{1}{\text{sec.}}$	R^2 %
1	3.25	180.0	19.33	8.953	16.968	84.62
		375.0	26.03	7.950	15.353	83.28
		750.0	26.30	8.633	15.334	84.30
		Average	23.89	8.512	15.885	
	5.75	180.0	12.35	10.044	15.383	88.75
		375.0	22.13	12.506	12.335	82.95
		750.0	14.93	9.478	14.911	81.78
		Average	16.47	10.676	14.210	
	8.25	180.0	4.41	10.523	15.061	88.57
		375.0	6.15	10.951	14.452	78.24
		750.0	6.46	10.659	12.977	85.20
		Average	5.67	10.711	14.163	
2	3.25	187.5	17.54	11.273	14.046	83.11
		375.0	23.25	13.187	13.870	86.90
		750.0	19.33	12.960	12.700	80.87
		Average	20.04	12.473	13.539	
	5.75	187.5	10.89	9.051	14.274	87.87
		375.0	11.08	8.779	12.825	89.93
		750.0	15.66	12.068	13.983	89.15
		Average	12.54	9.966	13.694	
	8.25	187.5	4.56	8.202	13.798	84.81
		375.0	7.46	11.846	11.680	92.34
		750.0	6.68	11.081	13.015	93.19
		Average	6.23	10.376	12.831	

Check for the Response Function

The response functions determined from the input-output data of the impulse tests were verified employing Equation 15. Essentially, this equation uses the impulse load input and the derived response function for a particular case. Computations result in the deflection function which was previously used to obtain the response function. Hence, the predicted deflection function should match the corresponding measured value. This checking procedure was applied to all the response functions as shown in Appendix A. The agreement was observed to be close to perfect. Typical plots are illustrated in Figures 36, 37 and 38. Table 13 shows the predicted and measured values for Series 1HI3 as a typical illustration.

Load-Independency of the Response Function

In this investigation, the response functions for almost all cases* were determined at three different peak stress levels, namely 14.93, 29.86 and 59.72 psi designated respectively, by the numbers 1, 2 and 3 in the data coding system. It was found that the response functions derived at these stress levels for any test condition, that is at the same surface course thickness, temperature and spatial location, did not change significantly. Comparisons of representative

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* The response functions for the 1LI1 Series were not determined because the deflection functions were not clearly recorded. Only the peak deflection values are included in the analysis of test results.

TABLE 13

TYPICAL PREDICTED AND MEASURED DEFLECTION FUNCTIONS

Series: 1HI3

Time sec.	Deflection Functions, inch ($\times 10^{-4}$)					
	Distance from Load Center, inches					
	3.25		5.75		8.25	
	Pred.	Meas.	Pred.	Meas.	Pred.	Meas.
0	0	0	0	0	0	0
0.02	0	0	0	0	0	0
0.04	0.90	0.90	0.80	0.80	0.40	0.40
0.06	15.00	15.00	6.40	6.40	2.24	2.24
0.08	17.10	17.10	9.60	9.60	3.44	3.44
0.10	19.50	19.50	10.40	10.40	3.88	3.88
0.12	20.40	20.40	10.72	10.72	4.08	4.08
0.14	21.00	21.00	11.04	11.04	4.12	4.12
0.16	21.15	21.15	11.20	11.20	4.16	4.16
0.18	21.30	21.30	11.52	11.52	4.24	4.24
0.20	21.60	21.60	11.68	11.68	4.28	4.28
0.22	21.75	21.75	11.84	11.84	4.32	4.32
0.24	12.00	12.00	4.80	4.80	3.20	3.20
0.26	4.50	4.50	2.80	2.80	1.60	1.60
0.28	2.40	2.40	2.08	2.08	1.12	1.12
0.30	2.10	2.10	1.76	1.76	0.80	0.80
0.32	1.80	1.80	1.60	1.60	0.76	0.76
0.34	1.65	1.65	1.52	1.52	0.72	0.72
0.36	1.50	1.50	1.44	1.44	0.64	0.64
0.38	1.20	1.20	1.36	1.36	0.60	0.60
0.40	1.05	1.05	1.20	1.20	0.56	0.56
0.42	0.90	0.90	1.12	1.12	0.52	0.52
0.44	0.90	0.90	1.04	1.04	0.48	0.48
0.46	0.90	0.90	0.96	0.96	0.44	0.44
0.48	0.90	0.90	0.88	0.88	0.44	0.44
0.50	0.90	0.90	0.88	0.88	0.44	0.44

plots of the response functions illustrated in Figures 39, 40, and 41 support this observation. Table 14 summarizes the first positive peak, the time of its occurrence, the first negative peak and the time of its occurrence for each of these response functions. It is evident from this table that these four parameters do not change significantly as the peak impulse load F_p is changed at any spatial location considered.

Inasmuch as the parameters in Equation 14 approximately represent the respective response function curve, the values of the α, β and γ parameters shown in Tables 10, 11 and 12 are examined to further demonstrate the load-independency of the response functions. It is seen from these tables that the α, β and γ parameters for each case do not vary appreciably as the magnitude of load changes. One exception with a distinct variability is evident in Table 10, and that is the magnitude of α for the one-inch surface when x is equal to 8.25 inches and the load is 750 lbs. It is possible that this discrepancy may be reduced if more parameters are included in the curve fitting process.

From the above discussion it is seen to be feasible to use the average values for α, β and γ in order to study the effect on these parameters of the other test variables. This is the subject of the next two sections.

TABLE 14

COMPARISON OF RESPONSE FUNCTIONS

OBTAINED FROM DIFFERENT LOAD MAGNITUDES

Surface Course Thickness: 1 inch Test Temperature: 100°F					
		First Positive Peak		First Negative Peak	
x	F_p	Magnitude	Time	Magnitude	Time
inches	lbs.	$\frac{\text{in.}}{\text{lb.-sec.}} \times 10^{-6}$	sec.	$\frac{\text{in.}}{\text{lb.-sec.}} \times 10^{-6}$	sec.
3.25	180.0	8.40	0.06	-3.32	0.20
	375.0	12.48	0.06	-5.25	0.22
	750.0	11.10	0.06	-4.37	0.22
5.75	180.0	5.19	0.04	-1.31	0.22
	375.0	6.46	0.04	-1.64	0.24
	750.0	6.01	0.04	-2.06	0.22
8.25	180.0	1.64	0.06	-0.38	0.22
	375.0	2.19	0.06	-0.76	0.22
	750.0	2.24	0.06	-0.54	0.24

Effect of Temperature on the Response Function

The parameters α, β and γ are, respectively, measures of the peak of the response function G(t) and of how G(t) varies with time. As such, α can be regarded as representing the stiffness characteristic of a pavement system, whereas β and γ reflect the damping characteristics of the system. Pavement materials having large α values will provide less resistance to imposed loads.

Referring to the curves of α versus temperature shown in Figure 42, it is readily apparent that temperature plays an important role in the response function of pavement systems. It was mentioned elsewhere that flexible pavement components, and asphaltic concrete in particular, are thermoplastic. They yield increased response for increases in temperature, other test variables being the same, and hence yield increased α values. This trend is indicated by the curves of Figure 42.

Examination of the parameters in Tables 10, 11 and 12 reveals that changes in β and γ are small as temperature changes. This is possibly due to the fact that no significant change is observed in the shapes of the response function curves. Furthermore, the slight variations in the values of these parameters are not critical since they appear, respectively, as exponential and sinusoidal functions in the response function model (Equation 14).

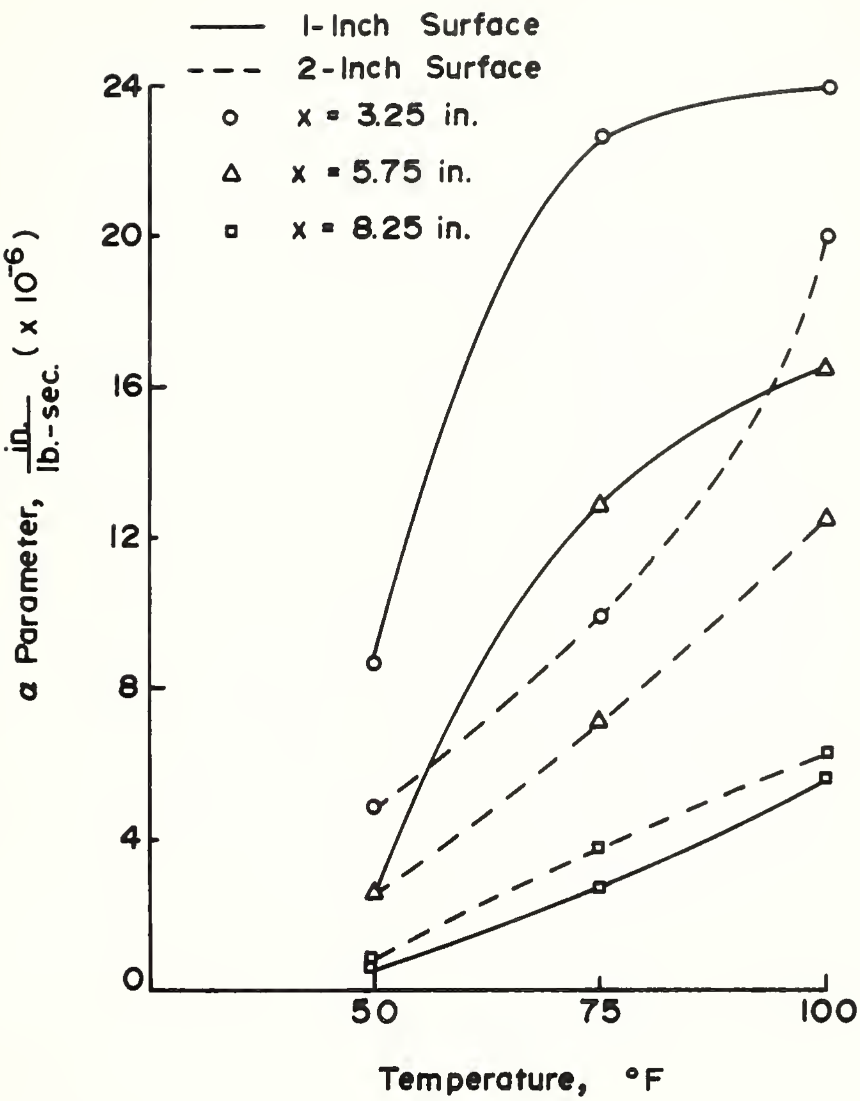

FIGURE 42 CURVES OF α PARAMETER VERSUS TEMPERATURE

Effects of Thickness and Spatial Location

From the graphical results shown in Figure 42, it is observed that at x equals 3.25 and 5.75 inches, the value of α decreases with increases in surface course thickness at all temperatures. However, at x equals 8.25 inches α increases with increases in thickness. This trend reflects the slab-like action of the system that a thicker pavement responds more at remote locations than a thinner one. Further examination of the figure shows that the curves indicate the interaction between surface course thickness and temperature. For the one-inch surface the rate of increase in α decreases with increases in temperature, whereas this rate is increased for the two-inch surface. It is recalled that similar observations were made previously regarding the effects of temperature and surface course thickness on the central deflection y_0. Obviously, the value of α decreases with increasing values of x, thus depicting the attenuation of the magnitude of α with spatial location.

Referring to Tables 10, 11 and 12, it is seen that the values of the parameters β and γ remain approximately constant with changes in the surface course and/or spatial location. This, again, is probably due to the similarity of the shapes of the response function curves.

Static Load Results

Static load deflections y(t) were calculated using Equation 23 of "Solution for Step Loading." Computations

were carried out employing the computer program STALOD listed in Appendix B. The predicted deflections and the corresponding measured values for all test series are summarized in Table 15. Typical plots are illustrated in Figures 43, 44 and 45. Close agreement in almost all cases is indicated in this table. It is very significant at this stage to recapitulate what has been done:

1. Pavement parameters were determined from impulse load tests.
2. The derived parameters were used in conjunction with a formulated theory to predict static load deflections for various test conditions of temperature, surface course thickness and spatial location.
3. The agreement between predicted and measured values was observed to be tolerable within experimental error.

Thus, the following conclusions, which will further be supported by the results and discussions of the next section, can be made:

1. The pavement parameters obtained in this investigation are indeed descriptors of pavement behavior.
2. Inasmuch as the two types of loads are different, one being dynamic and the other static, these parameters are independent of the type of input load.

TABLE 15

PREDICTED AND MEASURED STATIC LOAD DEFLECTIONS

Thick-ness	Temp.	F_o	y(t), inch ($\times 10^{-4}$)					
			Distance from Load Center, inches					
			3.25		5.75		8.25	
in.	°F	lbs.	Pred.	Meas.	Pred.	Meas.	Pred.	Meas.
1	50	375.0	3.00	4.50	0.72	1.20	0.26	0.32
		750.0	9.95	10.50	3.92	3.92	0.75	0.80
	75	187.5	4.38	4.16	1.98	2.08	0.43	0.48
		375.0	9.81	9.20	5.48	5.24	1.15	1.12
		750.0	20.36	20.70	10.90	10.40	2.83	2.80
	100	187.5	5.05	4.95	2.97	3.20	1.03	1.12
		375.0	13.51	12.15	8.54	8.16	2.77	2.92
		750.0	26.62	25.60	14.46	14.40	5.58	5.60
2	50	187.5	1.02	0.98	0.53	0.48	0.16	0.16
		375.0	1.58	1.80	0.84	0.88	0.26	0.27
		750.0	5.52	5.55	3.79	3.44	1.63	1.44
	75	187.5	2.23	2.06	1.65	1.60	0.93	0.84
		375.0	3.95	4.50	2.59	2.36	1.71	1.52
		750.0	11.33	11.25	7.36	7.04	3.38	3.36
	100	187.5	3.84	4.35	2.63	2.72	1.13	1.20
		375.0	9.33	9.00	5.19	5.12	2.88	2.64
		750.0	14.86	15.00	13.23	12.00	5.67	5.60

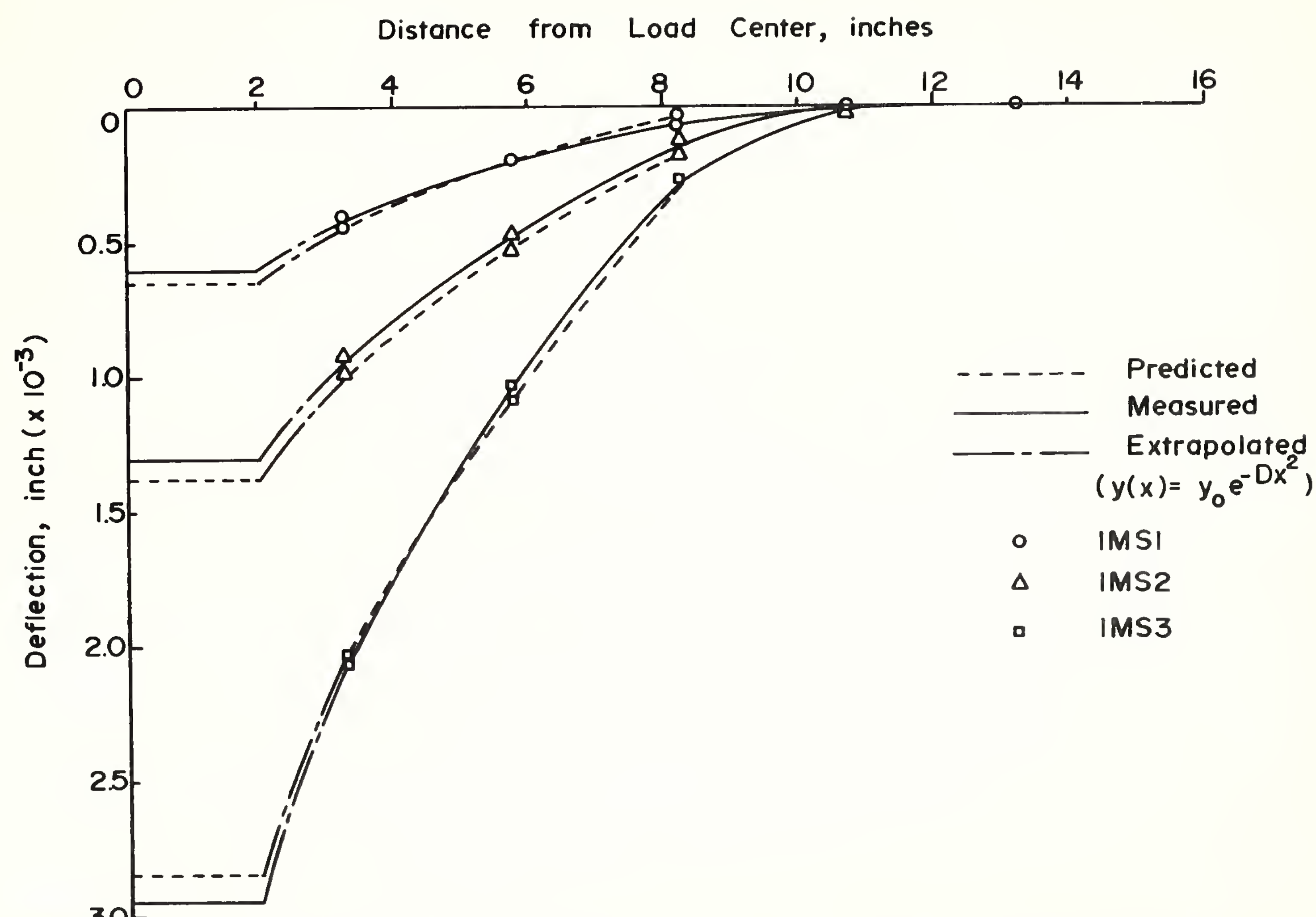

FIGURE 43 TYPICAL PREDICTED AND MEASURED STATIC LOAD DEFLECTIONS — 1-INCH SURFACE, 75°F

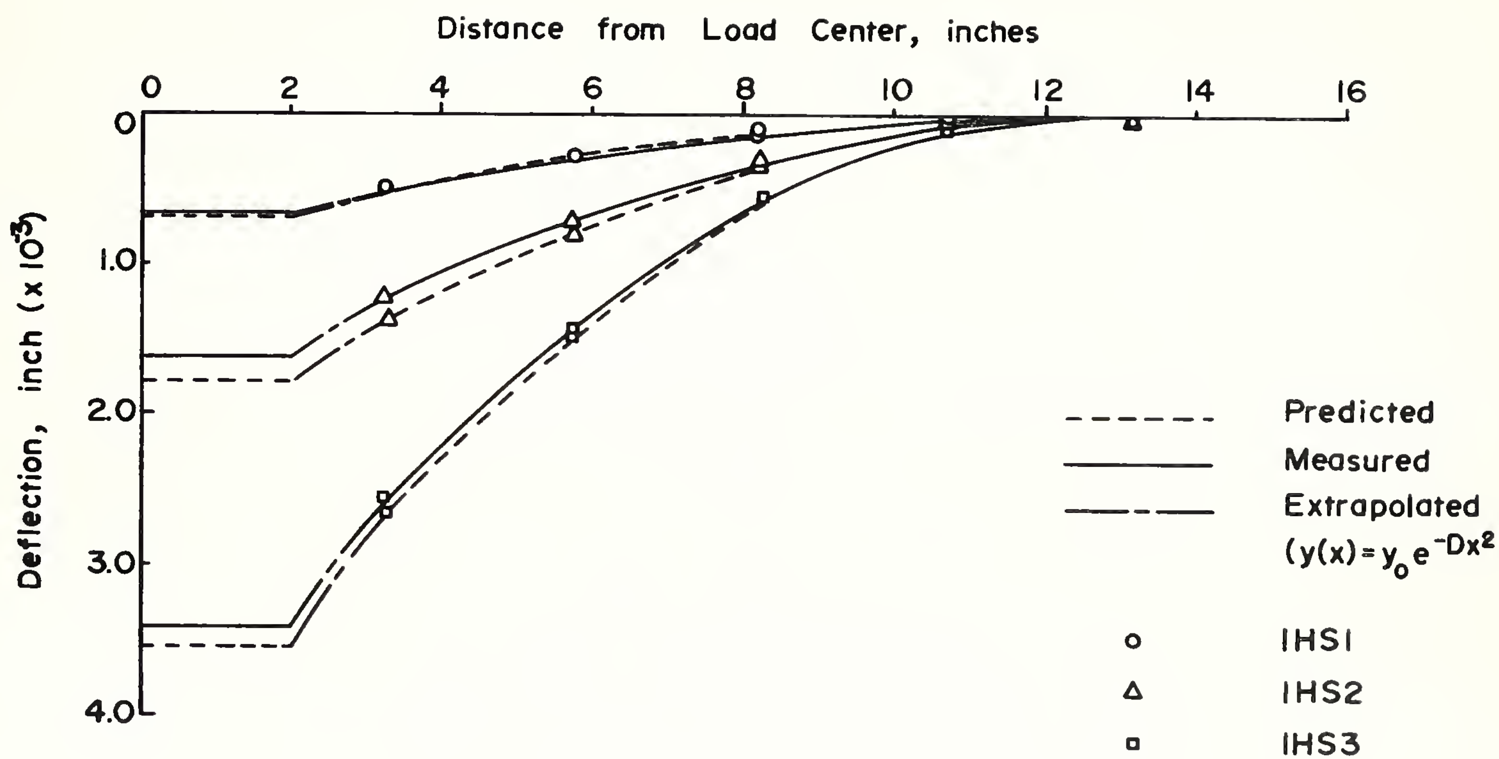

FIGURE 44 TYPICAL PREDICTED AND MEASURED STATIC LOAD DEFLECTIONS — 1-INCH SURFACE, 100°F

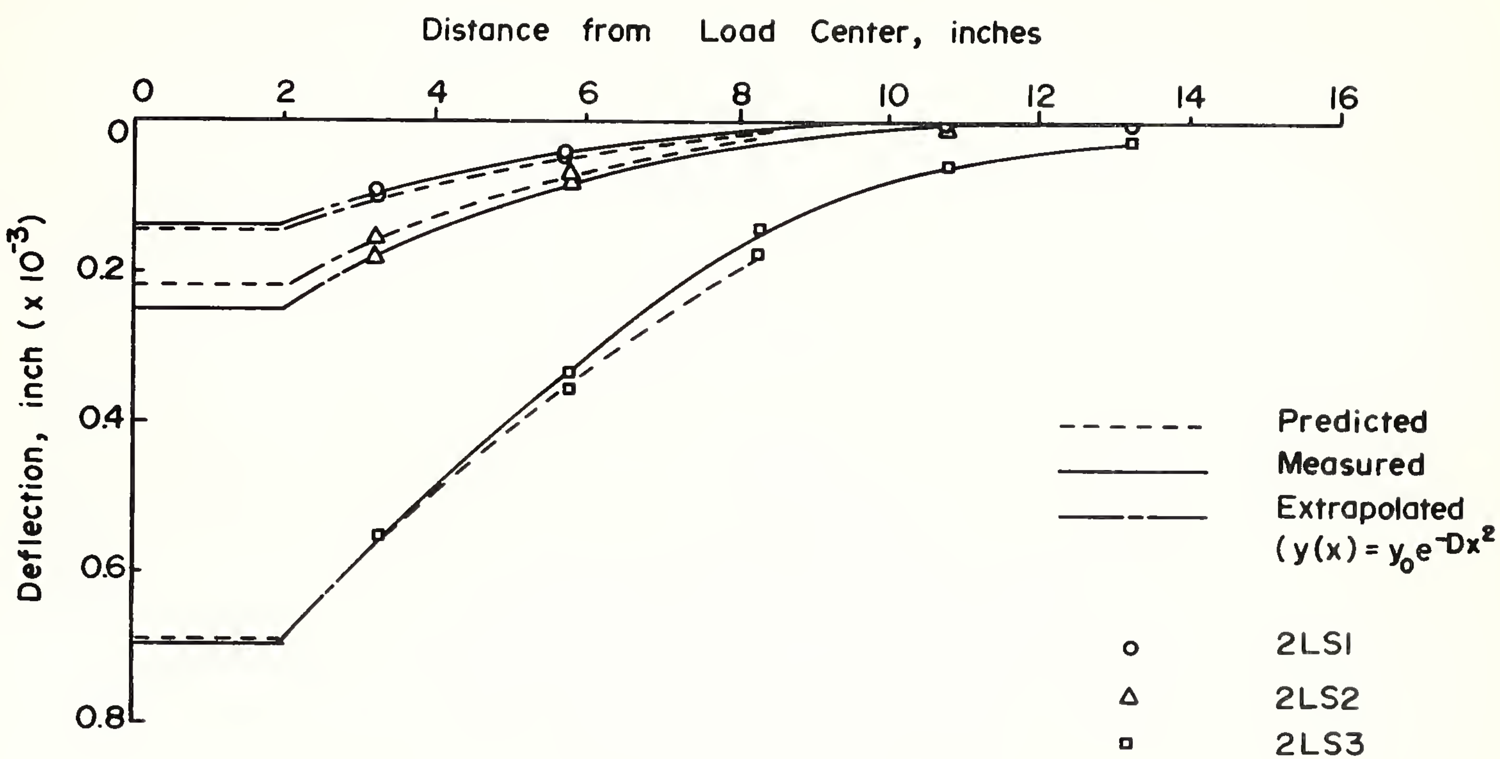

FIGURE 45 TYPICAL PREDICTED AND MEASURED STATIC LOAD DEFLECTIONS — 2-INCH SURFACE, 50°F

Repeated Load Results

Haversine load deflections were predicted employing Equations 28 and 30, respectively, for the total deformation $y_T(t)$ and the permanent deformation $y_P(t)$. Calculations were effected by the computer program REPLOD of Appendix C. Table 16 shows comparison of predicted and measured values of $y_T(t)$. Typical plots are shown in Figures 46, 47 and 48. The agreement is observed to be as good as that in the case of static loads. The results indicate that repeated load deflections, as well as static load deflections, can be predicted using the theory developed in this investigation.

Comparisons of the calculated deflections in Table 15 for the static loads and the corresponding values in Table 16 for the haversine loads show that the magnitudes are essentially the same in both cases. This is not surprising since it is easily verified that the repeated load equation (Equation 28) reduces to the static load equation (Equation 23) as the time t gets large. It appears that the time duration of 4 seconds used in haversine load tests is large enough to consider Equation 23 as the limiting case of Equation 28.

Regarding the measured values of the static load and repeated load deflections presented, respectively, in Tables 15 and 16, it is noticed that the deflections observed from repeated load tests are always less than those resulting from static load tests. This is due to the difference in

Repeated Load Results

Haversine load deflections were [illegible]

Equations 28 and 29, [illegible]

$y(t)$ and the [illegible]

were [illegible]

[illegible]

repeated [illegible]

15 and 16 [illegible]

repeated load tests [illegible]

from static load tests [illegible]

TABLE 16

PREDICTED AND MEASURED HAVERSINE LOAD DEFLECTIONS

Thick-ness	Temp.	F_o	$y_T(t)$, inch ($\times 10^{-4}$)					
			Distance from Load Center, inches					
			3.25		5.75		8.25	
in.	°F	lbs.	Pred.	Meas.	Pred.	Meas.	Pred.	Meas.
1	50	375.0	2.99	3.00	0.71	0.80	0.26	0.24
		750.0	9.94	9.15	3.81	3.36	0.75	0.68
	75	187.5	4.37	3.98	1.98	1.92	0.43	0.40
		375.0	9.80	9.08	5.48	5.04	1.15	0.96
		750.0	20.34	19.80	10.88	10.24	2.82	2.76
	100	187.5	5.04	4.32	2.97	2.52	1.03	0.96
		375.0	13.50	10.24	8.52	6.08	2.77	2.32
		750.0	26.61	24.80	14.45	13.12	5.57	4.88
2	50	187.5	1.02	0.94	0.53	0.48	0.16	0.16
		375.0	1.58	1.73	0.85	0.84	0.26	0.24
		750.0	5.52	4.95	3.79	3.40	1.63	1.40
	75	187.5	2.22	1.95	1.65	1.44	0.93	0.80
		375.0	3.95	3.90	2.59	2.24	1.71	1.44
		750.0	11.32	9.15	7.35	6.40	3.38	2.88
	100	187.5	3.84	3.75	2.63	2.48	1.13	1.04
		375.0	9.32	8.25	5.19	5.04	2.87	2.48
		750.0	14.84	14.63	13.22	11.04	5.66	5.04

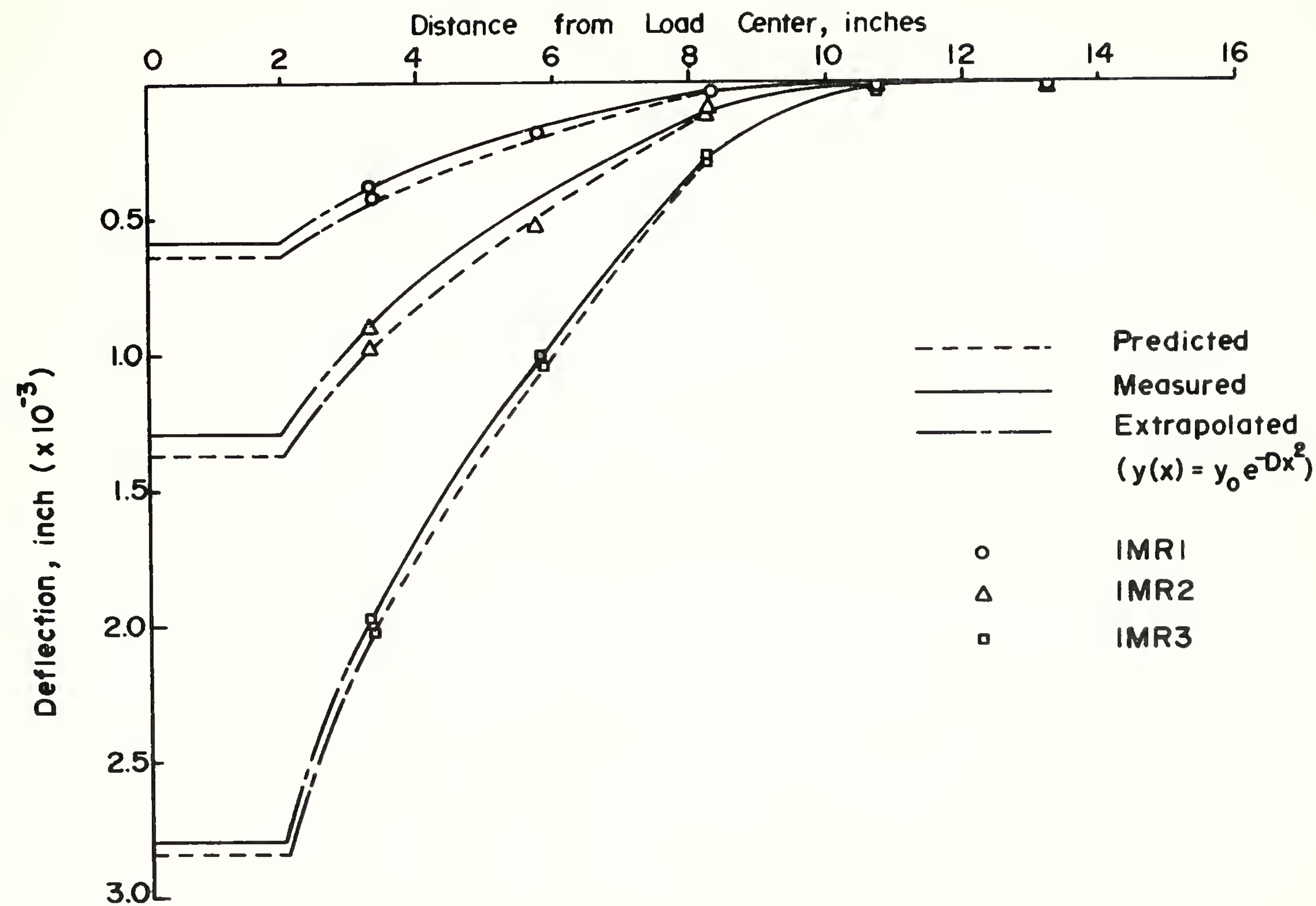

FIGURE 46 TYPICAL PREDICTED AND MEASURED REPEATED DEFLECTIONS — 1-INCH SURFACE, 75°F

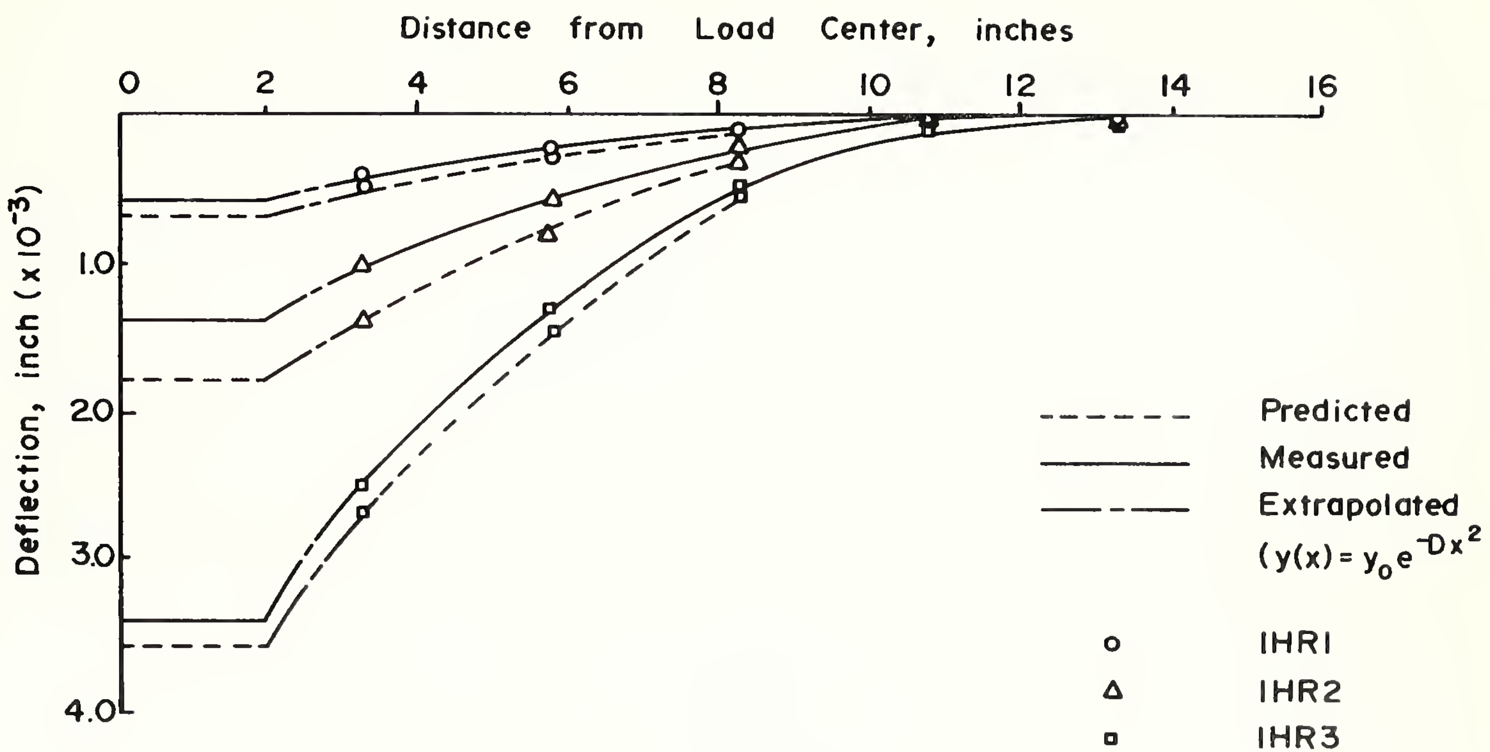

FIGURE 47 TYPICAL PREDICTED AND MEASURED REPEATED LOAD DEFLECTIONS—1-INCH SURFACE, 100°F

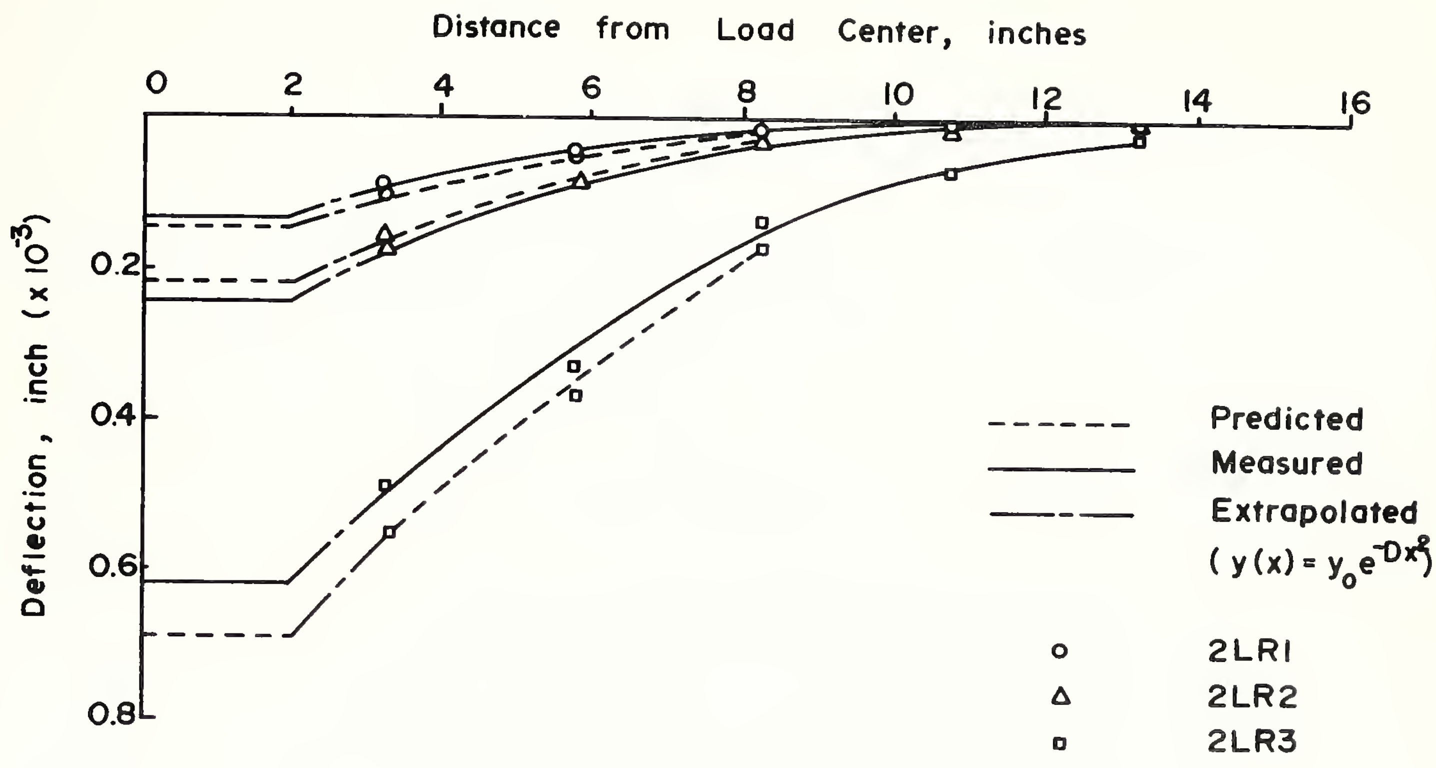

FIGURE 48 TYPICAL PREDICTED AND MEASURED REPEATED LOAD DEFLECTIONS — 2-INCH SURFACE, 50 °F

load durations which were, respectively, 4 seconds and a few minutes.

Both the calculated and measured values of the repeated load deflections did not vary with increasing number of load applications. From the theoretical point of view, the terms that would reflect this change are exponential (Equations 28 and 30). Because of the rapidly decaying feature of exponential functions, the contribution of these terms to the calculated deflections becomes negligible with increasing number of load applications. Experimentally, observed repeated load deflections at locations other than directly under the load result from the first few cycles and remain constant thereafter. This is indicated in the representative output traces of Figure 24. The observation is also substantiated by the results of Drennon and Kenis [62] obtained from laboratory tests of a full scale, three-layered flexible pavement.

The magnitudes of both calculated and measured values of repeated load permanent deformations $y_p(t)$ were too small (in the order of 10^{-5} inches or less) for any meaningful interpretations. Theoretically, it is evident from Equation 30 that $y_p(t)$ becomes zero in the limit as t increases. From the practical point of view, field observations of flexible pavement behavior indicate that rutting, which is mainly a measure of permanent or non-recoverable deformations, occurs directly under the wheel loads.

Corner Loading Test Results

A question arose whether the confinement of the model pavements in a box would have any effect on the center loading test results. Advanced indications of what might be expected were three-fold:

1. Previous laboratory studies dealing with pavements found edge effects to be negligible [58, 61].
2. Coating the interior sides of the box with spar varnish diluted with linseed oil would prevent adhesion of the layer materials to the box sides and thus would place no restriction on the vertical pavement response.
3. Deflections measured by LVDT no. 5 at 3 inches from the edge during center loading tests were small indicating that the response at the pavement edges would be small enough to minimize any concern about the effects of confinement.

Nonetheless, to gain more confidence in the over-all results, a series of corner loading tests was conducted on the model pavement with a two-inch surface, and at the most severe test conditions regarding edge effects. The two dynamic loads, namely impulse and cyclic, were employed at the 59.72 stress level. The test temperature was 50°F. These tests also had as another objective to see whether the response would vary much at different locations but equidistant from the center of the load. Thus, any non-uniformity in the preparation of the model pavements would be detected.

The test results of the corner loadings are compared in Table 17 with those of the center loadings. It appears from this table that there is reasonable agreement between the values measured at the inner sides of the plate during corner loading and the corresponding values from center loading. The corner loading results indicate that the magnitudes near the edge are slightly larger than the corresponding values recorded at the inner sides of the plate. This is probably due to the cantilever action that might have influenced the responses near the pavement edges. From the discussion of this section, it can be safely concluded that the edge effects were negligible and that the manner in which the model pavements were prepared is satisfactory.

TABLE 17

EFFECTS OF CONFINEMENT

Surface Course Thickness: 2 inches Test Temperature: 50°F Magnitude of Load: 750 lbs.					
x inches	Location*	Measured Deflections, inch (× 10^{-4})			
		Mode of Loading			
		Impulse		Haversine	
		Center Loading	Corner Loading	Center Loading	Corner Loading
3.25	Edge Side	4.88	4.73	4.95	5.03
5.75	Inner Side	3.36	3.28	3.40	3.44
	Edge Side	--	3.36	--	3.52
8.25	Inner Side	1.36	1.28	1.40	1.44
	Edge Side	--	1.44	--	1.52

* For corner loading tests. See Figure 18.

CONCLUSIONS

In the present investigation three-layer flexible pavement models utilizing one-inch and two-inch surface course thicknesses were tested. Impulse, static and repeated loads, of intensities 14.93, 29.86 and 59.72 psi, were applied at the center of the pavements. A few test series were also conducted at the corner. Test temperatures were 50°, 75° and 100°F. Based on the results and within the scope of this study, the conclusions are enumerated here. It should be recognized that these conclusions pertain to the materials and testing procedures used in this study. Justifiable extrapolation of the results should be made only upon further testing.

1. Transfer function theory is capable of predicting static or repeated load deflections of flexible pavements. The favorable agreement between predicted and measured values of the deflections in this study validates the hypothesis that the parameters in the response function are material descriptors which are independent of the type of load input.

2. The time-dependent behavior of a flexible pavement can be represented by a response function G(t) which is a function of time t. It is possible to obtain this function

from a single impulse test on the pavement. The response function is independent of the magnitude of the impulse load justifying the assumption of linearity.

3. The response function of a flexible highway pavement is of the form

$$G(t) = \alpha e^{-\beta t} \sin \gamma t$$

where α, β and γ are descriptive parameters in the response function.

4. Parameters in the response function are believed to be pavement properties that provide for better understanding of its behavior than those currently used.

5. Temperature, surface course thickness and spatial location have their respective influences on the response function. Increases in temperature increase the value of the α parameter in the response function, while increasing surface course thickness or the distance from the load center decreases the value of α. The β and γ parameters do not seem to be affected appreciably by the above factors.

6. The profile of peak deflections of a flexible highway pavement can be described by the equation

$$y(x) = y_0 e^{-Dx^2}$$

where $y(x)$ is the deflection at a distance x from the load center, y_0 is the central deflection and D is a constant reflecting the attenuation of the deflection profile with x.

7. The central deflection y_0 increases with increases in temperature, and decreases with increasing surface course

thickness. Increases in temperature (50°F - 100°F) or surface course thickness (1 inch and 2 inches) decrease the value of the parameter D.

8. Instrumentation and experimental procedures conducted were satisfactory for the test conditions of this investigation. The edge effects due to the confinement of the model pavements in the box are negligible with regard to the results of this study.

thickness. Increases in temperature [illegible]

surface course thickness [illegible]

value on the parameter [illegible]

[illegible]

SUGGESTIONS FOR FURTHER RESEARCH

The results of this project have been based on laboratory testing of flexible model pavements. It is realized that this is a first attempt in applying transfer function theory to highway pavements and more work needs to be done. The following research proposals are recommended as extensions of this investigation:

1. In this study, the response functions were approximated by the mathematical model

$$G(t) = \alpha e^{-\beta t} \sin \gamma t \qquad \text{Model (1)}$$

The following models could be used:

$$G(t) = \alpha e^{-\beta t} \sin \gamma t - \delta \sin \varepsilon t \qquad \text{Model (2)}$$

$$G(t) = \alpha e^{-\beta t} \sin \gamma t + \delta \cos \varepsilon t \qquad \text{Model (3)}$$

where $G(t)$ is the response function, t is time and α, β, γ, δ and ε are parameters.

Preliminary analysis of some of the data indicated that using Models 2 and 3 improved the squared correlation coefficient (R^2), as shown in Table 18. The theory of this investigation can be extended after replacing Model 1 by either Model 2 or 3.

TABLE 18

COMPARISON OF R^2 FOR DIFFERENT MODELS

Series	x inches	R^2, % Model (1)	Model (2)	Model (3)
1HI1	3.25	84.62	86.98	87.73
	5.75	88.75	91.38	91.85
	8.25	88.57	91.14	92.39
1HI2	3.25	83.23	90.26	86.89
	5.75	83.04	89.52	90.52
	8.25	78.24	87.10	86.94
1HI3	3.25	84.30	88.38	88.74
	5.75	81.78	88.11	87.53
	8.25	84.92	91.89	91.81
2HI1	3.25	83.11	85.24	89.42
	5.75	87.90	92.25	91.48
	8.25	84.81	91.40	88.70
2HI2	3.25	86.90	88.14	90.22
	5.75	89.93	95.07	93.36
	8.25	92.34	93.75	96.77
2HI3	3.25	80.87	83.23	89.71
	5.75	89.15	90.03	93.85
	8.25	93.19	94.78	96.38

2. The results of this investigation indicate feasibility of conducting field tests on highway pavements. The signatures from a truck travelling over a bump can be used to derive the response functions. Application of transfer function theory to evaluate airport pavements has just been completed [64].

The test program should cover a variety of construction types based on a large statistical sample to permit quantitative evaluation of the effects of the test factors involved. In the present study, the effects of variables were evaluated qualitatively.

3. The mathematical function obtained in this study to describe pavement deflection profiles might be correlated with failure in flexible pavements. Available data or deflections measured from planned field tests may be used for this purpose. By relating the slope of the deflected profile to the nature and extent of pavement failure, a rational method of evaluation could emerge.

4. The pavements in this investigation were tested in their entirety and, as such, isolation of the material properties for each layer was not possible. A research program that would allow determination of the component properties would include testing a single subgrade layer and successively building up to the testing of a complete pavement.

Limited experimental work with a 3-inch diameter plate in this study and the results of other investigators [70] indicated that the loaded area may have an effect on pavement properties. Variation of this factor may be incorporated in the laboratory testing program.

5. In this investigation, the measured output parameter was deflection, based on the premise that surface deflections are indicative of a flexible pavement's ability to withstand stresses produced within the pavement by imposed loads [71, 72]. However, this places no restriction on measuring other parameters, such as stress or strain. Inventory tests conducted in this study using strain gages showed that instruments with better resolution would be needed to record strain magnitudes of the order of 20 microns.

6. In the present study, pavement response was analyzed without resort to elastic or viscoelastic theory. However, individuals interested in extending the application of such theory can use the experimental technique of this investigation. By impulse testing material specimens, more reliable properties can be determined and used in conjunction with these theories.

7. The parameters determined in this investigation may be correlated and compared with those obtained by other researchers.

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APPENDICES

APPENDIX A

REDUCED IMPULSE DATA

Program

```
      PROGRAM MAIN(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=PUNCH,PLOT)
C
C     ******************************************************
C
C
C                 COMPUTER PROGRAM TO CALCULATE RESPONSE
C       FUNCTIONS AND PREDICT DEFLECTIONS AS A CHECK ON THE
C        RESPONSE FUNCTIONS
C
C
C     ******************************************************
C
C     DESCRIPTION OF VARIABLES USED
C     I         =IDENTIFICATION FOR DEFLECTION RESPONSE FUNCTIONS
C     J         =DATA SET NUMBER.
C     N         =NUMBER OF SEQUENCE VALUES IN THE DEFLECTION
C     RESPONSE FUNCTIONS.
C     IPLOT     =COMMAND FOR PLOT OF VARIABLES.
C     DT        =INCREMENT OF TIME BETWEEN EACH SEQUENCE VALUE
C        IN THE DEFLECTION RESPONSE FUNCTIONS.
C     DEFL(I,K)=ARRAY OF SEQUENCE VALUES FOR I-TH DEFLECTION
C      RESPONSE FUNCTION.
C     GT(I,K)   =ARRAY OF SEQUENCE VALUES FOR I-TH TIME
C     DEPENDENT TRANSFER FUNCTION.
C     GF(I,K)   =DUMMY VARIABLE FOR TEMPORARY STORAGE OF
C      SEQUENCE VALUES.
C     IPS(I)    =NUMBER OF SEQUENCE VALUES FOR PHASE SHIFT OF
          I-TH DEFLECTION RESPONSE FUNCTION.
C     GXX       =PEAK LOAD
C
      DIMENSION V1(40),V2(40)-V3(40),V4(40),V5(40),V6(40)
      DIMENSION XINP(40),XOUT(40),XG(40),Y(40),Z(40),W(6)
      DIMENSION DEFL(6,40),GF(6,40),GT(6,40),DE(6,40)
      COMMON/AXIS/ X(50)
      DIMENSION MONT(10)
      DIMENSION IPS(12),M912)
      DIMENSION PEAK(10),CONT(10),SPK(10)
      DIMENSION NAME(10)
      INTEGER CONT,BEGIN,END
      REAL MOD
      READ(5,100) BEGIN,END,N,DT,IPLOT,INEG,IMOD,ISMOTH
      N1=N
      DO 1000 J=BEGIN,END
      WRITE(6,208) J
      READ(5,106) (NAME(L),L=1,9)
  106 FORMAT(4X,9A8)
      WRITE(6,107) (NAME(L),L=1,9)
  107 FORMAT(1H1,////1X,9A8)
      read(5,101) IDATA,IFLAG,JDATA,JFLAG,KDATA,KFLAG,LDATA,
     1LFLAG,MDATA,
```

```
     2MFLAG,NDATA,NFLAG,(M(I),I=1,6),(IPS(I),I=1,6),GXX
      GO TO (29,700),IFLAG
   29 DO 30 I=1,6
   30 READ(5,102) (DEFL(I,K),K=1,N)
      WRITE(6,200)
      DO 261 K=1,N
  261 X(K)=FLOAT(K)*DT-DT
      DO 262 K=1,N
  262 WRITE(6,201) X(K),(DEFL(I,K),I=1,4)
  500 DO 501 I=1,6
      DO 501 K=1,N
  501 GF(I,K)=0.0
      IF(IFLAG.EQ.1) GF(2,NPHASE)=SPIKE
      DO 504 I=1,5
      IF(M(I).EQ.0)GO TO 504
      DO 502 K=1,N
      XINP(K)=DEFL(1,K)
  502 XOUT(K)=DEFL(I+1,K)
      CALL CONVLI(XINP,XOUT,XG,DT,N,GXX)
      DO 503 K=1,N
      GT(I+1,K)=XG(K)
  503 GF(I+1,K)=XG(K)
  504 CONTINUE
      WRITE(6,107) (NAME(L),L=1,9)
      WRITE(6,203)
 1572 DO 505 K=1,N
      X(K)=FLOAT(K)*DT-DT
      WRITE(6,202) X(K),(GF(I,K),I=2,4)
  505 CONTINUE
C1120 PUNCH
      IF(JFLAG.EQ. 0) GO TO 525
      DO 530 K=1,N
      V2(K)=0.0
      V3(K)=0.0
      V4(K)=0.0
      V5(K)=0.0
      V6(K)=0.0
  530 CONTINUE
C
C     .....DETERMINE SCALING PARAMETERS FOR RESPONSE FUNCTIONS
C
      DO 506 K=1,N
      V2(K)=GF(2,K)
      V3(K)=GF(3,K)
      V4(K)=GF(4,K)
      V5(K)=GF(5,K)
      V6(K)=GF(6,K)
  506 CONTINUE
      CALL SCALES(10.0,V2,N,1,V3,N,1,V4,N,1,V5,N,1,V6,N,1)
      CALL SCALE(x,10.0,N,1)
C
C     .....PLOT AXES AND ANNOTATION.....
C
```

```
      CALL AXIS(0.0,0.0,12HTIME,SECONDS,-12,10.0,0.0,X(N+1),
     1X(N+2),0)
      CALL AXIS(0.0,0.0,17HRESPONSE FUNCTION,17,10.0,90.0,
     1V2(N+1),V2(N+2),-1)
      CALL SYMBOL(3.5,10.0,0.2,18HRESPONSE FUNCTIONS,0.0,18)
      CALL SYMBOL(8.0,8.20,0.08,1,0.0,-1)
      CALL SYMBOL(8.1,8.14,0.08,11H-RESPONSE 1,0.0,11)
      CALL SYMBOL(8.0,8.1,0.08,2,0.0,-1)
      CALL SYMBOL(8.1,8.04,0.08,11H-RESPONSE 2,0.0,11)
      CALL SYMBOL(8.0,8.0,0.08,3,0.0,-1)
      CALL SYMBOL(8.1,7.94,0.08,11H-RESPONSE 3,0.0,11)
      CALL SYMBOL(8.0,7.9,0.08,4,0.0,-1)
      CALL SYMBOL(8.1,7.84,0.08,11H-RESPONSE 4,0.0,11)
      CALL SYMBOL(8.0,7.8,0.08,5,0.0,-1)
      CALL SYMBOL(8.1,7.74,0.08,11H-RESPONSE 5,0.0,11)
      CALL SYMBOL(8.0,7.7,0.08,6,0.0,-1)
      CALL SYMBOL(8.1,7.64,0.08,11H-RESPONSE 6,0.0,11)
C
C     .....PLOT RESPONSE FUNCTIONS.....
C
      DO 517 I=1,5
      IF(M(I).EQ.0) GO TO 517
      DO 509 K=1,N
  509 Z(K)=GF(I,K)
      Z(N+1)=V2(N+1)
      Z(N+2)=V2(N+2)
      CALL LINE(X,Z,N,1,2,I)
  517 CONTINUE
      CALL PLOT(18.0,0.0,-3)
  525 CONTINUE
C
  700 IF(MDATA.EQ.0) GO TO 702
  701 READ(5,102) (DEFL(1,K),K=1,N)
  702 IF(MFLAG.EQ.0) GO TO 704
      DO 703 I=2,4
  703 READ(5,102) (GT(I,K),K=1,N)
C     .....CONVERT STORED AND/OR INPUT DATA TO SUBROUTINE FORM
  704 DO 706 K=1,N
      GF(1,K)=DEFL(1,K)
  706 XINP(K)=DEFL(1,K)
      XINP(3)=10.*GXX
  707 DO 710 I=2,4
      DO 708 K=1,N
  708 XG(K)=GT(I,K)
      CALL CONVLE(XINP,XOUT,XG,DT,N)
      DO 709 K=1,N
  709 GF(I,K)=XOUT(K)
  710 CONTINUE
C     .....WRITE DEFLECTION RESPONSES.....
```

```
      WRITE(6,107) (NAME(L),L=1,9)
      WRITE(6,207)
      DO 711 K=1,N
  711 X(K)=FLOAT(K)*DT-DT
      DO 712 K=1,N
  712 WRITE(6,201) X(K),(GF(I,K),I=1,4)
 1001 N=N1
 1000 CONTINUE
      IF(IPLOT.NE.0) CALL PLOT(0,0,999)
C
C     ****************************************************
C
C     .....FORMAT STATEMENTS.....
C
C     ****************************************************
C
  100 FORMAT(3I3,F10.5,4I1)
  101 FORMAT(18I1,6I2,F10.5)
  102 FORMAT(8F10.5)
  103 FORMAT(I2)
  108 FORMAT(8F10.8)
  200 FORMAT(///25X,'RECORD OF RAW DATA',///8X,'TIME',2X,
     1'LOAD,LBS.',3X,'D E F L E C T I O N S,IN.',//7X,'(SEC)'
     2,4X,'X=0',6X,'X=3.25',5X,'X=5.75',5X,'X=8.25'///)
  201 FORMAT((1X,F11.2,F8.1,3(F11.6))
  202 FORMAT(1X,F12.3,3(F14.8))
  203 FORMAT(///,8X,'RESPONSE FUNCTIONS FROM IMPLICIT CONVO'
     1'LUTION'//8X,'TIME',7X,'R E S P O N S E   F U N C T I '
     2'O N S'//7X,'(SEC)',7X,'X=3.25',8X,'X=5.75',8X,'X=8.25'
     3///)
  204 FORMAT(1X,F12.6,12X,5F12.6)
  206 FORMAT(1X,2F12.6)
  207 FORMAT(///,9X,'DEFLECTIONS FROM RESPONSE FUNCTIONS AN'
     1'D LOAD',///8X,'TIME',2X,'LOAD,LBS.',3X,'D E F L E C '
     2'T I O N S,IN.'//7X,'(SEC)',4X,'X=0',6X,'X=3.25',5X,
     3'X=5.75',5X,'X=8.25'///)
  208 FORMAT(1H1,1X/////////////////////////1X,'
     *                               DATA SET NUMBER',2X,I2)
  210 FORMAT(21(8E10.3/),7E10.3)
  214 FORMAT(2F10.8)
C
C     ====================================================
C
      STOP
      END
C     ====================================================
C
C     SUBROUTINE CONVLI
```

```
C     ======================================================
C     PURPOSE
C        TO COMPUTE THE VECTOR OF VALUES OF
C            XG FROM CONVOLUTION EQUATION XINP*XG=XOUT
C            FOR GIVEN XINP(I) AND XOUT(I)
C     ======================================================
C       DISCRIPTION OF PARAMETERS
C        XINP-INPUT VECTOR OF VALUES
C        XOUT-OUTPUT VECTOR OF VALUES AFTER CONVOLUTION
C        XG-VECTOR OF VALUES OF CONVOLUTION FUNCTION
C        DT-INCREMENT OF THE SUMMATION POINTS
C        N-NUMBER OF SUBINTERVAL PLUS 1.
C     SUBROUTINES AND FUNCTION SUBPROGRAMS NEEDED
C        NONE
C     METHOD
C        1.  SET THE VALUE OF 'TEST'. THEN TEST THE VALUES
C      OF XINP(I) TO
C            SEE WHETHER XINP(I) IS GREATER THAN TEST OR NOT
C        2.  IF MAGNITUDE OF XINP(K) IS GREATER THAN 'TEST'
C     THEN MAKE UP K MORE VALUES OF XOUT(I) AND K MORE
C        SUMMATION EQUATIONS
C        3.  CONSIDER THE SUMMATION EQUATION  WHICH IS
C     TRANSFERED FROM INTRGRATION EQUATION OF CONVOLUTION.
C        4.  BY FORWARD SUBSTITUTION METHOD TO
C            SOLVE FOR XG(I),I=1,...,N.
C     .......................................................
      SUBROUTINE CONVLI(XINP,XOUT,XG,DT,N,GXX)
C     .......................................................
      DIMENSION XINP(1),XOUT(1),XG(1)
      DIMENSION Z(200),Y(200),W(200)
      A=0.
      TEST=.000001
      DO 150 I=1,N
  150 XG(I)=0.0
      DO 151 K=1,N
      IF(ABS(XINP(K)).LT.TEST) GO TO 151
      L=K
      GO TO 152
  151 CONTINUE
  152 IN=L-1
      IM=N+L-1
      IF(IN.EQ.0) GO TO 156
      XDOT=XOUT(N)/FLOAT(IN)
      DO 153 K=1,IN
  153 XOUT(N+K)=XOUT(N)-XDOT*FLOAT(K)
  156 CONTINUE
      DO 155 I=L,IM
      II=I-1
      DO 154 K=L,II
```

```
  154 A=A+XG(K-L+1)*XINP(I-K+L)
      XINP(L)=10.*GXX
      XG(I-L+1)=(XOUT(I)-A*DT)/(XINP(L)*DT)
  155 A=0.0
      RETURN
      END
C     ///////////////////////////////////////////////////////
C     SUBROUTINE CONVLE
C     .......................................................
C     PURPOSE
C        TO COMPUTE THE VECTOR OF VALUES OF XOUT(I) FOR GIVEN
C        VECTORS...XINP(I) AND XG(I) BY USING CONVOLUTION
C        (I.E. CONVOLUTE XINP(I) AND XG(I) BY SUMMATION METHOD)
C       DISCRIPTION OF PARAMETERS
C        XINP-INPUT VECTOR OF VALUES
C        XOUT-OUTPUT VECTOR OF VALUES AFTER CONVOLUTION
C        XG-VECTOR OF VALUES OF CONVOLUTION FUNCTION
C        DT-INCREMENT OF THE SUMMATION POINTS
C        N-NUMBER OF SUBINTERVAL PLUS 1.
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS NEEDED
C        NONE
C     METHOD
C        TRANSFER THE INTEGRATION FORMULA OF CONVOLUTION INTO
C        FINITE SUMMATION EQATIONS WITH SUBINTERVALS OF LENGTH DT
C    AND NUMBER N-1. COMPUTE XOUT(I) BY GIVEN VALUES OF XINP
C        AND XG DIRECTLY
C     .......................................................
      SUBROUTINE CONVLE(XINP,XOUT,XG,DT,N)
      DIMENSION XINP(1),XOUT(1),XG(1)
      DO 100 I=1,N
      XOUT(I)=0.
      DO 150 J=1,I
      XOUT(I)=XOUT(I)+XG(J)*XINP(I-J+1)*DT
  150 CONTINUE
  100 CONTINUE
      RETURN
      END
```

Reduced Data

The reduction of the impulse test data was implemented using the program of this Appendix. Each test series consists of three tables. These are: input and outputs (raw data), response functions and predicted deflection functions, in that order.

1LI2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000060	.000013	.000004
.06	200.0	.000150	.000016	.000008
.08	375.0	.000173	.000035	.000012
.10	375.0	.000195	.000045	.000016
.12	375.0	.000218	.000061	.000020
.14	375.0	.000225	.000067	.000024
.16	375.0	.000240	.000070	.000028
.18	375.0	.000244	.000074	.000032
.20	375.0	.000248	.000077	.000032
.22	0.0	.000165	.000048	.000020
.24	0.0	.000098	.000032	.000012
.26	0.0	.000068	.000013	.000008
.28	0.0	.000049	.000010	.000004
.30	0.0	.000045	.000006	0.000000
.32	0.0	.000038	.000005	0.000000
.34	0.0	.000030	.000003	0.000000
.36	0.0	.000023	.000003	0.000000
.38	0.0	.000015	.000003	0.000000
.40	0.0	.000008	.000003	0.000000
.42	0.0	.000008	.000003	0.000000
.44	0.0	.000008	.000003	0.000000
.46	0.0	.000008	.000003	0.000000
.48	0.0	.000008	.000003	0.000000
.50	0.0	.000008	.000003	0.000000

1LI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000080	.00000017	.00000005
.020	.00000196	.00000020	.00000010
.040	.00000212	.00000044	.00000015
.060	.00000221	.00000054	.00000019
.080	.00000230	.00000070	.00000023
.100	.00000217	.00000072	.00000026
.120	.00000215	.00000069	.00000029
.140	.00000198	.00000067	.00000031
.160	.00000183	.00000064	.00000028
.180	.00000061	.00000021	.00000010
.200	-.00000020	-.00000003	-.00000002
.220	-.00000041	-.00000024	-.00000006
.240	-.00000041	-.00000021	-.00000009
.260	-.00000019	-.00000018	-.00000011
.280	-.00000004	-.00000010	-.00000008
.300	.00000008	-.00000004	-.00000004
.320	.00000018	.00000003	-.00000000
.340	.00000025	.00000010	.00000003
.360	.00000019	.00000011	.00000004
.380	.00000015	.00000010	.00000003
.400	.00000009	.00000006	.00000002
.420	.00000004	.00000003	.00000001
.440	.00000001	.00000001	-.00000000
.460	.00000001	-.00000000	-.00000001
.480	-.00000004	-.00000002	-.00000001
.500	-.00000007	-.00000004	-.00000001

1LI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000060	.000013	.000004
.06	200.0	.000150	.000016	.000008
.08	375.0	.000173	.000035	.000012
.10	375.0	.000195	.000045	.000016
.12	375.0	.000218	.000061	.000020
.14	375.0	.000225	.000067	.000024
.16	375.0	.000240	.000070	.000028
.18	375.0	.000244	.000074	.000032
.20	375.0	.000248	.000077	.000032
.22	0.0	.000165	.000048	.000020
.24	0.0	.000098	.000032	.000012
.26	0.0	.000068	.000013	.000008
.28	0.0	.000049	.000010	.000004
.30	0.0	.000045	.000006	-.000000
.32	0.0	.000038	.000005	.000000
.34	0.0	.000030	.000003	0.000000
.36	0.0	.000023	.000003	-.000000
.38	0.0	.000015	.000003	-.000000
.40	0.0	.000008	.000003	-.000000
.42	0.0	.000008	.000003	-.000000
.44	0.0	.000008	.000003	.000000
.46	0.0	.000008	.000003	0.000000
.48	0.0	.000008	.000003	.000000
.50	0.0	.000008	.000003	.000000

1LI3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000075	.000032	.000010
.06	720.0	.000525	.000192	.000022
.08	750.0	.000675	.000256	.000051
.10	750.0	.000705	.000296	.000059
.12	750.0	.000720	.000300	.000061
.14	750.0	.000735	.000300	.000062
.16	750.0	.000750	.000304	.000064
.18	750.0	.000758	.000304	.000066
.20	750.0	.000765	.000304	.000067
.22	0.0	.000450	.000152	.000048
.24	0.0	.000165	.000032	.000019
.26	0.0	.000105	.000024	.000010
.28	0.0	.000090	.000016	.000006
.30	0.0	.000075	.000012	.000003
.32	0.0	.000060	.000008	.000002
.34	0.0	.000045	.000008	.000002
.36	0.0	.000030	.000008	.000002
.38	0.0	.000023	.000008	.000002
.40	0.0	.000015	.000008	.000002
.42	0.0	.000015	.000008	.000002
.44	0.0	.000015	.000008	.000002
.46	0.0	.000015	.000008	.000002
.48	0.0	.000015	.000008	.000002
.50	0.0	.000015	.000008	.000002

1LI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000050	.00000021	.00000007
.020	.00000345	.00000126	.00000014
.040	.00000412	.00000156	.00000032
.060	.00000391	.00000168	.00000034
.080	.00000362	.00000154	.00000032
.100	.00000335	.00000138	.00000030
.120	.00000312	.00000127	.00000028
.140	.00000286	.00000114	.00000026
.160	.00000262	.00000103	.00000024
.180	.00000031	-.00000007	.00000010
.200	-.00000129	-.00000074	-.00000009
.220	-.00000115	-.00000057	-.00000011
.240	-.00000075	-.00000039	-.00000009
.260	-.00000041	-.00000023	-.00000007
.280	-.00000013	-.00000009	-.00000004
.300	.00000009	.00000004	-.00000001
.320	.00000027	.00000015	.00000002
.340	.00000046	.00000024	.00000004
.360	.00000039	.00000021	.00000005
.380	.00000022	.00000012	.00000003
.400	.00000009	.00000005	.00000002
.420	.00000000	.00000000	.00000001
.440	-.00000004	-.00000002	.00000000
.460	-.00000005	-.00000003	-.00000000
.480	-.00000008	-.00000005	-.00000001
.500	-.00000010	-.00000005	-.00000001

1LI3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000075	.000032	.000010
.06	720.0	.000525	.000192	.000022
.08	750.0	.000675	.000256	.000051
.10	750.0	.000705	.000296	.000059
.12	750.0	.000720	.000300	.000061
.14	750.0	.000735	.000300	.000062
.16	750.0	.000750	.000304	.000064
.18	750.0	.000758	.000304	.000066
.20	750.0	.000765	.000304	.000067
.22	0.0	.000450	.000152	.000048
.24	0.0	.000165	.000032	.000019
.26	0.0	.000105	.000024	.000010
.28	0.0	.000090	.000016	.000006
.30	0.0	.000075	.000012	.000003
.32	0.0	.000060	.000008	.000002
.34	0.0	.000045	.000008	.000002
.36	0.0	.000030	.000008	.000002
.38	0.0	.000023	.000008	.000002
.40	0.0	.000015	.000008	.000002
.42	0.0	.000015	.000008	.000002
.44	0.0	.000015	.000008	.000002
.46	0.0	.000015	.000008	.000002
.48	0.0	.000015	.000008	.000002
.50	0.0	.000015	.000008	.000002

1MI1

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000045	.000040	.000008
.06	150.0	.000259	.000104	.000019
.08	187.5	.000270	.000120	.000022
.10	187.5	.000285	.000128	.000029
.12	187.5	.000293	.000136	.000032
.14	187.5	.000300	.000140	.000034
.16	187.5	.000308	.000144	.000035
.18	187.5	.000315	.000148	.000037
.20	187.5	.000323	.000152	.000038
.22	0.0	.000195	.000080	.000029
.24	0.0	.000135	.000056	.000019
.26	0.0	.000098	.000048	.000016
.28	0.0	.000045	.000040	.000013
.30	0.0	.000038	.000032	.000010
.32	0.0	.000030	.000024	.000006
.34	0.0	.000023	.000016	.000003
.36	0.0	.000015	.000008	.000003
.38	0.0	.000008	.000008	.000003
.40	0.0	.000008	.000008	.000003
.42	0.0	.000008	.000008	.000003
.44	0.0	.000008	.000008	.000003
.46	0.0	.000008	.000008	.000003
.48	0.0	.000008	.000008	.000003
.50	0.0	.000008	.000008	.000003

1MI1

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000120	.00000107	.00000021
.020	.00000681	.00000269	.00000049
.040	.00000654	.00000288	.00000053
.060	.00000628	.00000281	.00000066
.080	.00000586	.00000274	.00000068
.100	.00000545	.00000257	.00000066
.120	.00000511	.00000242	.00000062
.140	.00000478	.00000228	.00000061
.160	.00000451	.00000215	.00000058
.180	.00000076	.00000012	.00000030
.200	-.00000031	-.00000030	.00000005
.220	-.00000064	-.00000020	.00000001
.240	-.00000136	-.00000011	-.00000000
.260	-.00000084	-.00000004	-.00000002
.280	-.00000042	.00000001	-.00000006
.300	-.00000004	.00000004	-.00000007
.320	.00000023	.00000005	-.00000000
.340	.00000048	.00000026	.00000006
.360	.00000051	.00000025	.00000008
.380	.00000043	.00000019	.00000008
.400	.00000032	.00000015	.00000007
.420	.00000015	.00000013	.00000007
.440	.00000005	.00000011	.00000006
.460	.00000000	.00000010	.00000005
.480	-.00000011	-.00000001	-.00000001
.500	-.00000019	-.00000012	-.00000005

1MI1

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000045	.000040	.000008
.06	150.0	.000259	.000104	.000019
.08	187.5	.000270	.000120	.000022
.10	187.5	.000285	.000128	.000029
.12	187.5	.000293	.000136	.000032
.14	187.5	.000300	.000140	.000034
.16	187.5	.000308	.000144	.000035
.18	187.5	.000315	.000148	.000037
.20	187.5	.000323	.000152	.000038
.22	0.0	.000195	.000080	.000029
.24	0.0	.000135	.000056	.000019
.26	0.0	.000098	.000048	.000016
.28	0.0	.000045	.000040	.000013
.30	0.0	.000038	.000032	.000010
.32	0.0	.000030	.000024	.000006
.34	0.0	.000023	.000016	.000003
.36	0.0	.000015	.000008	.000003
.38	0.0	.000008	.000008	.000003
.40	0.0	.000008	.000008	.000003
.42	0.0	.000008	.000008	.000003
.44	0.0	.000008	.000008	.000003
.46	0.0	.000008	.000008	.000003
.48	0.0	.000008	.000008	.000003
.50	0.0	.000008	.000008	.000003

1MI2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000120	.000064	.000003
.06	300.0	.000608	.000320	.000066
.08	375.0	.000638	.000336	.000072
.10	375.0	.000653	.000352	.000077
.12	375.0	.000668	.000368	.000080
.14	375.0	.000690	.000376	.000082
.16	375.0	.000713	.000384	.000083
.18	375.0	.000735	.000392	.000085
.20	375.0	.000750	.000400	.000086
.22	0.0	.000413	.000256	.000058
.24	0.0	.000293	.000176	.000026
.26	0.0	.000128	.000112	.000016
.28	0.0	.000098	.000080	.000013
.30	0.0	.000060	.000056	.000010
.32	0.0	.000045	.000040	.000006
.34	0.0	.000030	.000024	.000005
.36	0.0	.000023	.000016	.000003
.38	0.0	.000015	.000016	.000003
.40	0.0	.000015	.000016	.000003
.42	0.0	.000015	.000016	.000003
.44	0.0	.000015	.000016	.000003
.46	0.0	.000015	.000016	.000003
.48	0.0	.000015	.000016	.000003
.50	0.0	.000015	.000016	.000003

1MI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000160	.00000085	.00000004
.020	.00000798	.00000420	.00000088
.040	.00000771	.00000406	.00000089
.060	.00000713	.00000386	.00000086
.080	.00000661	.00000369	.00000082
.100	.00000623	.00000342	.00000076
.120	.00000591	.00000318	.00000070
.140	.00000560	.00000296	.00000065
.160	.00000524	.00000277	.00000060
.180	.00000037	.00000065	.00000017
.200	-.00000057	-.00000010	-.00000019
.220	-.00000196	-.00000055	-.00000023
.240	-.00000148	-.00000055	-.00000016
.260	-.00000116	-.00000044	-.00000010
.280	-.00000062	-.00000027	-.00000007
.300	-.00000016	-.00000013	-.00000000
.320	.00000034	.00000007	.00000004
.340	.00000073	.00000035	.00000009
.360	.00000070	.00000038	.00000010
.380	.00000057	.00000034	.00000007
.400	.00000032	.00000025	.00000004
.420	.00000013	.00000016	.00000002
.440	.00000000	.00000010	.00000001
.460	-.00000006	.00000006	.00000000
.480	-.00000017	-.00000006	-.00000002
.500	-.00000023	-.00000016	-.00000003

1MI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000120	.000064	.000003
.06	300.0	.000608	.000320	.000066
.08	375.0	.000638	.000336	.000072
.10	375.0	.000653	.000352	.000077
.12	375.0	.000668	.000368	.000080
.14	375.0	.000690	.000376	.000082
.16	375.0	.000713	.000384	.000083
.18	375.0	.000735	.000392	.000085
.20	375.0	.000750	.000400	.000086
.22	0.0	.000413	.000256	.000058
.24	0.0	.000293	.000176	.000026
.26	0.0	.000128	.000112	.000016
.28	0.0	.000098	.000080	.000013
.30	0.0	.000060	.000056	.000010
.32	0.0	.000045	.000040	.000006
.34	0.0	.000030	.000024	.000005
.36	0.0	.000023	.000016	.000003
.38	0.0	.000015	.000016	.000003
.40	0.0	.000015	.000016	.000003
.42	0.0	.000015	.000016	.000003
.44	0.0	.000015	.000016	.000003
.46	0.0	.000015	.000016	.000003
.48	0.0	.000015	.000016	.000003
.50	0.0	.000015	.000016	.000003

1MI3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000240	.000128	.000008
.06	600.0	.001200	.000624	.000164
.08	750.0	.001260	.000656	.000172
.10	750.0	.001320	.000688	.000180
.12	750.0	.001380	.000720	.000184
.14	750.0	.001425	.000752	.000188
.16	750.0	.001455	.000768	.000192
.18	750.0	.001485	.000784	.000196
.20	750.0	.001500	.000800	.000200
.22	0.0	.001110	.000576	.000136
.24	0.0	.000570	.000416	.000060
.26	0.0	.000360	.000256	.000044
.28	0.0	.000270	.000192	.000036
.30	0.0	.000210	.000128	.000032
.32	0.0	.000150	.000096	.000024
.34	0.0	.000120	.000064	.000016
.36	0.0	.000090	.000048	.000008
.38	0.0	.000060	.000032	.000008
.40	0.0	.000045	.000032	.000008
.42	0.0	.000030	.000032	.000008
.44	0.0	.000030	.000032	.000008
.46	0.0	.000030	.000032	.000008
.48	0.0	.000030	.000032	.000008
.50	0.0	.000030	.000032	.000008

IMI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000160	.00000085	.00000005
.020	.00000787	.00000409	.00000109
.040	.00000761	.00000396	.00000105
.060	.00000724	.00000378	.00000100
.080	.00000691	.00000361	.00000093
.100	.00000651	.00000346	.00000086
.120	.00000606	.00000321	.00000080
.140	.00000564	.00000299	.00000074
.160	.00000517	.00000280	.00000070
.180	.00000220	.00000111	.00000020
.200	-.00000089	.00000030	-.00000022
.220	-.00000150	-.00000041	-.00000021
.240	-.00000124	-.00000044	-.00000014
.260	-.00000082	-.00000046	-.00000006
.280	-.00000048	-.00000028	-.00000002
.300	-.00000002	-.00000014	.00000001
.320	.00000036	.00000007	.00000003
.340	.00000065	.00000024	.00000010
.360	.00000071	.00000033	.00000011
.380	.00000045	.00000033	.00000008
.400	.00000025	.00000025	.00000005
.420	.00000010	.00000018	.00000003
.440	.00000000	.00000012	.00000002
.460	-.00000005	.00000008	.00000001
.480	-.00000015	-.00000005	-.00000001
.500	-.00000020	-.00000015	-.00000004

1MT3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000240	.000128	.000008
.06	600.0	.001200	.000624	.000164
.08	750.0	.001260	.000656	.000172
.10	750.0	.001320	.000688	.000180
.12	750.0	.001380	.000720	.000184
.14	750.0	.001425	.000752	.000188
.16	750.0	.001455	.000768	.000192
.18	750.0	.001485	.000784	.000196
.20	750.0	.001500	.000800	.000200
.22	0.0	.001110	.000576	.000136
.24	0.0	.000570	.000416	.000060
.26	0.0	.000360	.000256	.000044
.28	0.0	.000270	.000192	.000036
.30	0.0	.000210	.000128	.000032
.32	0.0	.000150	.000096	.000024
.34	0.0	.000120	.000064	.000016
.36	0.0	.000090	.000048	.000008
.38	0.0	.000060	.000032	.000008
.40	0.0	.000045	.000032	.000008
.42	0.0	.000030	.000032	.000008
.44	0.0	.000030	.000032	.000008
.46	0.0	.000030	.000032	.000008
.48	0.0	.000030	.000032	.000008
.50	0.0	.000030	.000032	.000008

1HT1

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000022	.000016	.000006
.06	170.0	.000285	.000124	.000051
.08	180.0	.000315	.000200	.000064
.10	180.0	.000360	.000208	.000070
.12	180.0	.000367	.000212	.000072
.14	180.0	.000375	.000212	.000074
.16	180.0	.000390	.000216	.000075
.18	180.0	.000398	.000220	.000077
.20	180.0	.000405	.000224	.000078
.22	0.0	.000180	.000168	.000051
.24	0.0	.000060	.000064	.000026
.26	0.0	.000030	.000040	.000016
.28	0.0	.000022	.000024	.000010
.30	0.0	.000015	.000020	.000008
.32	0.0	.000007	.000016	.000006
.34	0.0	.000007	.000012	.000005
.36	0.0	.000007	.000008	.000003
.38	0.0	.000007	.000008	.000003
.40	0.0	.000007	.000008	.000003
.42	0.0	.000007	.000008	.000003
.44	0.0	.000007	.000008	.000003
.46	0.0	.000007	.000008	.000003
.48	0.0	.000007	.000008	.000003
.50	0.0	.000007	.000008	.000003

1HI1

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	R E S P O N S E F U N C T I O N S X=3.25	X=5.75	X=8.25
0.000	.00000062	.00000044	.00000018
.020	.00000786	.00000340	.00000141
.040	.00000795	.00000519	.00000163
.060	.00000840	.00000490	.00000164
.080	.00000777	.00000452	.00000152
.100	.00000720	.00000407	.00000142
.120	.00000689	.00000377	.00000132
.140	.00000641	.00000350	.00000123
.160	.00000598	.00000326	.00000115
.180	-.00000081	.00000142	.00000030
.200	-.00000332	-.00000128	-.00000031
.220	-.00000304	-.00000131	-.00000038
.240	-.00000210	-.00000114	-.00000036
.260	-.00000132	-.00000068	-.00000022
.280	-.00000067	-.00000031	-.00000010
.300	.00000009	-.00000001	.00000000
.320	.00000073	.00000023	.00000008
.340	.00000126	.00000053	.00000019
.360	.00000105	.00000062	.00000020
.380	.00000061	.00000043	.00000015
.400	.00000025	.00000026	.00000010
.420	.00000001	.00000012	.00000005
.440	-.00000012	.00000004	.00000002
.460	-.00000018	.00000000	.00000001
.480	-.00000026	-.00000011	-.00000003
.500	-.00000026	-.00000019	-.00000007

1HI1

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD, LBS. X=0	D E F L E C T I O N S, IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000022	.000016	.000006
.06	170.0	.000285	.000124	.000051
.08	180.0	.000315	.000200	.000064
.10	180.0	.000360	.000208	.000070
.12	180.0	.000367	.000212	.000072
.14	180.0	.000375	.000212	.000074
.16	180.0	.000390	.000216	.000075
.18	180.0	.000397	.000220	.000077
.20	180.0	.000405	.000224	.000078
.22	0.0	.000180	.000168	.000051
.24	0.0	.000060	.000064	.000026
.26	0.0	.000030	.000040	.000016
.28	0.0	.000023	.000024	.000010
.30	0.0	.000015	.000020	.000008
.32	0.0	.000007	.000016	.000006
.34	0.0	.000008	.000012	.000005
.36	0.0	.000007	.000008	.000003
.38	0.0	.000007	.000008	.000003
.40	0.0	.000007	.000008	.000003
.42	0.0	.000007	.000008	.000003
.44	0.0	.000007	.000008	.000003
.46	0.0	.000007	.000008	.000003
.48	0.0	.000008	.000008	.000003
.50	0.0	.000008	.000008	.000003

1HT2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000030	.000040	.000016
.06	300.0	.000488	.000400	.000128
.08	375.0	.000825	.000520	.000176
.10	375.0	.001050	.000560	.000192
.12	375.0	.001095	.000576	.000198
.14	375.0	.001110	.000584	.000200
.16	375.0	.001125	.000592	.000203
.18	375.0	.001140	.000600	.000208
.20	375.0	.001155	.000608	.000210
.22	375.0	.001170	.000616	.000211
.24	0.0	.000600	.000440	.000080
.26	0.0	.000210	.000216	.000048
.28	0.0	.000150	.000144	.000045
.30	0.0	.000135	.000120	.000042
.32	0.0	.000128	.000112	.000040
.34	0.0	.000120	.000108	.000038
.36	0.0	.000105	.000104	.000035
.38	0.0	.000090	.000100	.000034
.40	0.0	.000083	.000096	.000032
.42	0.0	.000075	.000092	.000032
.44	0.0	.000075	.000088	.000032
.46	0.0	.000075	.000084	.000032
.48	0.0	.000075	.000080	.000032
.50	0.0	.000075	.000080	.000032

1HI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000040	.00000053	.00000021
.020	.00000647	.00000529	.00000169
.040	.00001044	.00000646	.00000219
.060	.00001248	.00000637	.00000219
.080	.00001187	.00000594	.00000206
.100	.00001087	.00000545	.00000187
.120	.00000996	.00000500	.00000172
.140	.00000915	.00000460	.00000161
.160	.00000842	.00000424	.00000148
.180	.00000776	.00000391	.00000134
.200	-.00000059	.00000122	-.00000052
.220	-.00000525	-.00000141	-.00000076
.240	-.00000457	-.00000164	-.00000051
.260	-.00000305	-.00000116	-.00000028
.280	-.00000162	-.00000055	-.00000007
.300	-.00000045	.00000001	.00000010
.320	.00000041	.00000047	.00000023
.340	.00000110	.00000083	.00000036
.360	.00000176	.00000113	.00000045
.380	.00000226	.00000136	.00000054
.400	.00000199	.00000130	.00000043
.420	.00000126	.00000097	.00000031
.440	.00000066	.00000065	.00000023
.460	.00000028	.00000046	.00000017
.480	-.00000042	-.00000018	-.00000006
.500	-.00000094	-.00000070	-.00000027

1HI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000030	.000040	.000016
.06	300.0	.000488	.000400	.000128
.08	375.0	.000825	.000520	.000176
.10	375.0	.001050	.000560	.000192
.12	375.0	.001095	.000576	.000198
.14	375.0	.001110	.000584	.000200
.16	375.0	.001125	.000592	.000203
.18	375.0	.001140	.000600	.000208
.20	375.0	.001155	.000608	.000210
.22	375.0	.001170	.000616	.000211
.24	0.0	.000600	.000440	.000080
.26	0.0	.000210	.000216	.000048
.28	0.0	.000150	.000144	.000045
.30	0.0	.000135	.000120	.000042
.32	0.0	.000128	.000112	.000040
.34	0.0	.000120	.000108	.000038
.36	0.0	.000105	.000104	.000035
.38	0.0	.000090	.000100	.000034
.40	0.0	.000083	.000096	.000032
.42	0.0	.000075	.000092	.000032
.44	0.0	.000075	.000088	.000032
.46	0.0	.000075	.000084	.000032
.48	0.0	.000075	.000080	.000032
.50	0.0	.000075	.000080	.000032

1HI3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	350.0	.000090	.000080	.000040
.06	600.0	.001500	.000640	.000224
.08	750.0	.001710	.000960	.000344
.10	750.0	.001950	.001040	.000388
.12	750.0	.002040	.001072	.000408
.14	750.0	.002100	.001104	.000412
.16	750.0	.002115	.001120	.000416
.18	750.0	.002130	.001152	.000424
.20	750.0	.002160	.001168	.000428
.22	750.0	.002175	.001184	.000432
.24	0.0	.001200	.000480	.000320
.26	0.0	.000450	.000280	.000160
.28	0.0	.000240	.000208	.000112
.30	0.0	.000210	.000176	.000080
.32	0.0	.000180	.000160	.000076
.34	0.0	.000165	.000152	.000072
.36	0.0	.000150	.000144	.000064
.38	0.0	.000120	.000136	.000060
.40	0.0	.000105	.000120	.000056
.42	0.0	.000090	.000112	.000052
.44	0.0	.000090	.000104	.000048
.46	0.0	.000090	.000096	.000044
.48	0.0	.000090	.000088	.000044
.50	0.0	.000090	.000088	.000044

1HI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000060	.00000053	.00000027
.020	.00000995	.00000422	.00000147
.040	.00001054	.00000601	.00000215
.060	.00001110	.00000598	.00000224
.080	.00001060	.00000559	.00000215
.100	.00000993	.00000524	.00000196
.120	.00000903	.00000481	.00000179
.140	.00000820	.00000454	.00000166
.160	.00000757	.00000418	.00000152
.180	.00000690	.00000387	.00000139
.200	-.00000024	-.00000117	.00000053
.220	-.00000437	-.00000206	-.00000046
.240	-.00000436	-.00000175	-.00000054
.260	-.00000301	-.00000119	-.00000048
.280	-.00000182	-.00000060	-.00000024
.300	-.00000073	-.00000006	-.00000004
.320	.00000017	.00000038	.00000009
.340	.00000079	.00000075	.00000023
.360	.00000138	.00000100	.00000033
.380	.00000185	.00000124	.00000041
.400	.00000165	.00000095	.00000040
.420	.00000104	.00000059	.00000029
.440	.00000049	.00000029	.00000020
.460	.00000013	.00000014	.00000013
.480	-.00000037	-.00000023	-.00000005
.500	-.00000072	-.00000052	-.00000020

1HI3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	350.0	.000090	.000080	.000040
.06	600.0	.001500	.000640	.000224
.08	750.0	.001710	.000960	.000344
.10	750.0	.001950	.001040	.000388
.12	750.0	.002040	.001072	.000408
.14	750.0	.002100	.001104	.000412
.16	750.0	.002115	.001120	.000416
.18	750.0	.002130	.001152	.000424
.20	750.0	.002160	.001168	.000428
.22	750.0	.002175	.001184	.000432
.24	0.0	.001200	.000480	.000320
.26	0.0	.000450	.000280	.000160
.28	0.0	.000240	.000208	.000112
.30	0.0	.000210	.000176	.000080
.32	0.0	.000180	.000160	.000076
.34	0.0	.000165	.000152	.000072
.36	0.0	.000150	.000144	.000064
.38	0.0	.000120	.000136	.000060
.40	0.0	.000105	.000120	.000056
.42	0.0	.000090	.000112	.000052
.44	0.0	.000090	.000104	.000048
.46	0.0	.000090	.000096	.000044
.48	0.0	.000090	.000088	.000044
.50	0.0	.000090	.000088	.000044

2LT1

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000011	.000010	.000003
.06	150.0	.000045	.000032	.000006
.08	175.0	.000064	.000034	.000008
.10	187.5	.000068	.000035	.000010
.12	187.5	.000068	.000037	.000011
.14	187.5	.000071	.000038	.000013
.16	187.5	.000071	.000040	.000014
.18	187.5	.000075	.000042	.000016
.20	187.5	.000075	.000042	.000018
.22	0.0	.000045	.000029	.000008
.24	0.0	.000038	.000016	.000006
.26	0.0	.000030	.000010	.000005
.28	0.0	.000023	.000006	.000003
.30	0.0	.000015	.000005	.000002
.32	0.0	.000011	.000003	.000002
.34	0.0	.000008	.000002	.000002
.36	0.0	.000004	.000002	.000002
.38	0.0	.000004	.000002	.000002
.40	0.0	.000004	.000002	.000002
.42	0.0	.000004	.000002	.000002
.44	0.0	.000004	.000002	.000002
.46	0.0	.000004	.000002	.000002
.48	0.0	.000004	.000002	.000002
.50	0.0	.000004	.000002	.000002

2LT1

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME	R E S P O N S E	F U N C T I O N S	
(SEC)	X=3.25	X=5.75	X=8.25
0.000	.00000029	.00000027	.00000008
.020	.00000118	.00000083	.00000015
.040	.00000159	.00000082	.00000019
.060	.00000155	.00000076	.00000023
.080	.00000139	.00000074	.00000023
.100	.00000133	.00000069	.00000026
.120	.00000120	.00000067	.00000026
.140	.00000118	.00000066	.00000029
.160	.00000106	.00000059	.00000032
.180	.00000018	.00000021	.00000003
.200	.00000008	-.00000008	-.00000002
.220	.00000001	-.00000016	-.00000003
.240	-.00000003	-.00000018	-.00000006
.260	-.00000010	-.00000011	-.00000005
.280	-.00000007	-.00000008	-.00000002
.300	-.00000002	-.00000003	.00000001
.320	-.00000001	.00000004	.00000004
.340	.00000010	.00000009	.00000007
.360	.00000011	.00000011	.00000006
.380	.00000011	.00000009	.00000005
.400	.00000010	.00000006	.00000005
.420	.00000009	.00000004	.00000004
.440	.00000007	.00000002	.00000003
.460	.00000005	.00000001	.00000002
.480	-.00000001	-.00000002	-.00000001
.500	-.00000006	-.00000004	-.00000003

2LT1

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000011	.000010	.000003
.06	150.0	.000045	.000032	.000006
.08	175.0	.000064	.000034	.000008
.10	187.5	.000068	.000035	.000010
.12	187.5	.000068	.000037	.000011
.14	187.5	.000071	.000038	.000013
.16	187.5	.000071	.000040	.000014
.18	187.5	.000075	.000042	.000016
.20	187.5	.000075	.000042	.000018
.22	0.0	.000045	.000029	.000008
.24	0.0	.000038	.000016	.000006
.26	0.0	.000030	.000010	.000005
.28	0.0	.000023	.000006	.000003
.30	0.0	.000015	.000005	.000002
.32	0.0	.000011	.000003	.000002
.34	0.0	.000008	.000002	.000002
.36	0.0	.000004	.000002	.000002
.38	0.0	.000004	.000002	.000002
.40	0.0	.000004	.000002	.000002
.42	0.0	.000004	.000002	.000002
.44	0.0	.000004	.000002	.000002
.46	0.0	.000004	.000002	.000002
.48	0.0	.000004	.000002	.000002
.50	0.0	.000004	.000002	.000002

2LI2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000023	.000006	.000005
.06	350.0	.000045	.000010	.000006
.08	375.0	.000090	.000038	.000010
.10	375.0	.000113	.000054	.000016
.12	375.0	.000124	.000070	.000021
.14	375.0	.000128	.000080	.000024
.16	375.0	.000131	.000083	.000026
.18	375.0	.000139	.000085	.000027
.20	375.0	.000143	.000086	.000029
.22	0.0	.000090	.000067	.000022
.24	0.0	.000083	.000054	.000019
.26	0.0	.000045	.000026	.000008
.28	0.0	.000030	.000019	.000005
.30	0.0	.000026	.000013	.000003
.32	0.0	.000023	.000010	.000002
.34	0.0	.000019	.000008	.000002
.36	0.0	.000015	.000006	.000002
.38	0.0	.000011	.000005	.000002
.40	0.0	.000008	.000003	.000002
.42	0.0	.000008	.000002	.000002
.44	0.0	.000008	.000002	.000002
.46	0.0	.000008	.000002	.000002
.48	0.0	.000008	.000002	.000002
.50	0.0	.000008	.000002	.000002

2LI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME	R E S P O N S E	F U N C T I O N S	
(SEC)	X=3.25	X=5.75	X=8.25
0.000	.00000031	.00000008	.00000007
.020	.00000057	.00000013	.00000007
.040	.00000112	.00000049	.00000012
.060	.00000131	.00000065	.00000019
.080	.00000133	.00000080	.00000024
.100	.00000125	.00000086	.00000025
.120	.00000117	.00000081	.00000025
.140	.00000116	.00000076	.00000024
.160	.00000109	.00000069	.00000024
.180	.00000031	.00000038	.00000013
.200	.00000024	.00000018	.00000009
.220	-.00000018	-.00000017	-.00000006
.240	-.00000024	-.00000018	-.00000007
.260	-.00000013	-.00000016	-.00000007
.280	-.00000003	-.00000010	-.00000005
.300	.00000003	-.00000003	-.00000002
.320	.00000009	.00000002	.00000001
.340	.00000014	.00000007	.00000003
.360	.00000012	.00000008	.00000004
.380	.00000013	.00000007	.00000005
.400	.00000010	.00000005	.00000004
.420	.00000006	.00000003	.00000002
.440	.00000004	.00000001	.00000002
.460	.00000004	-.00000000	.00000001
.480	-.00000002	-.00000002	-.00000001
.500	-.00000006	-.00000003	-.00000002

2LI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000023	.000006	.000005
.06	350.0	.000045	.000010	.000006
.08	375.0	.000090	.000038	.000010
.10	375.0	.000113	.000054	.000016
.12	375.0	.000124	.000070	.000021
.14	375.0	.000128	.000080	.000024
.16	375.0	.000131	.000083	.000026
.18	375.0	.000139	.000085	.000027
.20	375.0	.000143	.000086	.000029
.22	0.0	.000090	.000067	.000022
.24	0.0	.000083	.000054	.000019
.26	0.0	.000045	.000026	.000008
.28	0.0	.000030	.000019	.000005
.30	0.0	.000026	.000013	.000003
.32	0.0	.000023	.000010	.000002
.34	0.0	.000019	.000008	.000002
.36	0.0	.000015	.000006	.000002
.38	0.0	.000011	.000005	.000002
.40	0.0	.000008	.000003	.000002
.42	0.0	.000008	.000002	.000002
.44	0.0	.000008	.000002	.000002
.46	0.0	.000008	.000002	.000002
.48	0.0	.000008	.000002	.000002
.50	0.0	.000008	.000002	.000002

2LT3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000075	.000024	.000040
.06	700.0	.000150	.000088	.000080
.08	750.0	.000375	.000248	.000120
.10	750.0	.000435	.000304	.000124
.12	750.0	.000443	.000324	.000128
.14	750.0	.000450	.000328	.000132
.16	750.0	.000465	.000328	.000132
.18	750.0	.000480	.000332	.000136
.20	750.0	.000488	.000336	.000136
.22	0.0	.000225	.000248	.000080
.24	0.0	.000105	.000064	.000024
.26	0.0	.000075	.000020	.000008
.28	0.0	.000060	.000012	.000004
.30	0.0	.000045	.000008	.000004
.32	0.0	.000038	.000004	.000004
.34	0.0	.000030	.000004	.000004
.36	0.0	.000023	.000004	.000004
.38	0.0	.000015	.000004	.000004
.40	0.0	.000015	.000004	.000004
.42	0.0	.000015	.000004	.000004
.44	0.0	.000015	.000004	.000004
.46	0.0	.000015	.000004	.000004
.48	0.0	.000015	.000004	.000004
.50	0.0	.000015	.000004	.000004

2LI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000050	.00000016	.00000027
.020	.00000095	.00000057	.00000051
.040	.00000236	.00000158	.00000073
.060	.00000253	.00000181	.00000068
.080	.00000234	.00000176	.00000064
.100	.00000215	.00000161	.00000060
.120	.00000203	.00000145	.00000054
.140	.00000193	.00000133	.00000051
.160	.00000179	.00000122	.00000046
.180	-.00000010	.00000053	.00000007
.200	-.00000080	-.00000070	-.00000026
.220	-.00000069	-.00000077	-.00000027
.240	-.00000047	-.00000057	-.00000020
.260	-.00000029	-.00000036	-.00000012
.280	-.00000009	-.00000019	-.00000005
.300	.00000007	-.00000002	.00000001
.320	.00000021	.00000011	.00000006
.340	.00000032	.00000022	.00000010
.360	.00000028	.00000026	.00000010
.380	.00000017	.00000016	.00000006
.400	.00000008	.00000007	.00000003
.420	.00000003	.00000000	.00000001
.440	-.00000001	-.00000003	-.00000001
.460	-.00000001	-.00000005	-.00000001
.480	-.00000006	-.00000006	-.00000002
.500	-.00000008	-.00000006	-.00000003

2LT3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000075	.000024	.000040
.06	700.0	.000150	.000088	.000080
.08	750.0	.000375	.000248	.000120
.10	750.0	.000435	.000304	.000124
.12	750.0	.000443	.000324	.000128
.14	750.0	.000450	.000328	.000132
.16	750.0	.000465	.000328	.000132
.18	750.0	.000480	.000332	.000136
.20	750.0	.000488	.000336	.000136
.22	0.0	.000225	.000248	.000080
.24	0.0	.000105	.000064	.000024
.26	0.0	.000075	.000020	.000008
.28	0.0	.000060	.000012	.000004
.30	0.0	.000045	.000008	.000004
.32	0.0	.000038	.000004	.000004
.34	0.0	.000030	.000004	.000004
.36	0.0	.000023	.000004	.000004
.38	0.0	.000015	.000004	.000004
.40	0.0	.000015	.000004	.000004
.42	0.0	.000015	.000004	.000004
.44	0.0	.000015	.000004	.000004
.46	0.0	.000015	.000004	.000004
.48	0.0	.000015	.000004	.000004
.50	0.0	.000015	.000004	.000004

2MI1

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000015	.000012	.000005
.06	175.0	.000101	.000072	.000043
.08	187.5	.000135	.000108	.000058
.10	187.5	.000157	.000120	.000066
.12	187.5	.000167	.000124	.000069
.14	187.5	.000180	.000128	.000070
.16	187.5	.000184	.000132	.000070
.18	187.5	.000188	.000136	.000072
.20	187.5	.000191	.000140	.000074
.22	0.0	.000135	.000108	.000058
.24	0.0	.000101	.000072	.000043
.26	0.0	.000067	.000032	.000029
.28	0.0	.000030	.000024	.000013
.30	0.0	.000022	.000016	.000010
.32	0.0	.000015	.000008	.000006
.34	0.0	.000007	.000004	.000003
.36	0.0	.000007	.000004	.000003
.38	0.0	.000007	.000004	.000003
.40	0.0	.000007	.000004	.000003
.42	0.0	.000007	.000004	.000003
.44	0.0	.000007	.000004	.000003
.46	0.0	.000007	.000004	.000003
.48	0.0	.000007	.000004	.000003
.50	0.0	.000007	.000004	.000003

2MI1

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000040	.00000032	.00000013
.020	.00000266	.00000189	.00000114
.040	.00000331	.00000267	.00000142
.060	.00000358	.00000273	.00000149
.080	.00000348	.00000256	.00000143
.100	.00000348	.00000241	.00000133
.120	.00000323	.00000228	.00000119
.140	.00000301	.00000216	.00000112
.160	.00000281	.00000205	.00000105
.180	.00000106	.00000102	.00000053
.200	.00000031	.00000014	.00000020
.220	-.00000029	-.00000068	-.00000006
.240	-.00000091	-.00000056	-.00000034
.260	-.00000068	-.00000046	-.00000025
.280	-.00000046	-.00000038	-.00000017
.300	-.00000029	-.00000027	-.00000012
.320	.00000004	.00000002	.00000000
.340	.00000032	.00000022	.00000011
.360	.00000040	.00000030	.00000015
.380	.00000039	.00000029	.00000016
.400	.00000032	.00000019	.00000013
.420	.00000020	.00000011	.00000009
.440	.00000011	.00000006	.00000005
.460	.00000005	.00000001	.00000003
.480	-.00000008	-.00000007	-.00000003
.500	-.00000017	-.00000011	-.00000007

2MI1

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000015	.000012	.000005
.06	175.0	.000101	.000072	.000043
.08	187.5	.000135	.000108	.000058
.10	187.5	.000157	.000120	.000066
.12	187.5	.000167	.000124	.000069
.14	187.5	.000180	.000128	.000070
.16	187.5	.000184	.000132	.000070
.18	187.5	.000187	.000136	.000072
.20	187.5	.000191	.000140	.000074
.22	0.0	.000135	.000108	.000058
.24	0.0	.000101	.000072	.000043
.26	0.0	.000067	.000032	.000029
.28	0.0	.000030	.000024	.000013
.30	0.0	.000023	.000016	.000010
.32	0.0	.000015	.000008	.000006
.34	0.0	.000008	.000004	.000003
.36	0.0	.000007	.000004	.000003
.38	0.0	.000007	.000004	.000003
.40	0.0	.000007	.000004	.000003
.42	0.0	.000007	.000004	.000003
.44	0.0	.000007	.000004	.000003
.46	0.0	.000007	.000004	.000003
.48	0.0	.000007	.000004	.000003
.50	0.0	.000007	.000004	.000003

2MI2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000015	.000008	.000008
.06	350.0	.000135	.000072	.000072
.08	375.0	.000203	.000108	.000108
.10	375.0	.000270	.000176	.000124
.12	375.0	.000330	.000212	.000132
.14	375.0	.000345	.000248	.000136
.16	375.0	.000360	.000268	.000140
.18	375.0	.000367	.000272	.000144
.20	375.0	.000375	.000276	.000144
.22	0.0	.000270	.000176	.000108
.24	0.0	.000172	.000108	.000072
.26	0.0	.000135	.000072	.000032
.28	0.0	.000060	.000032	.000024
.30	0.0	.000045	.000024	.000016
.32	0.0	.000030	.000016	.000008
.34	0.0	.000015	.000008	.000004
.36	0.0	.000015	.000008	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2MI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000020	.00000011	.00000011
.020	.00000178	.00000095	.00000095
.040	.00000251	.00000134	.00000134
.060	.00000317	.00000212	.00000142
.080	.00000365	.00000239	.00000139
.100	.00000349	.00000263	.00000130
.120	.00000334	.00000264	.00000122
.140	.00000311	.00000243	.00000115
.160	.00000289	.00000224	.00000104
.180	.00000122	.00000069	.00000046
.200	-.00000003	-.00000020	.00000003
.220	-.00000029	-.00000053	-.00000038
.240	-.00000094	-.00000080	-.00000031
.260	-.00000069	-.00000059	-.00000024
.280	-.00000047	-.00000038	-.00000019
.300	-.00000028	-.00000018	-.00000011
.320	.00000006	.00000008	.00000002
.340	.00000034	.00000030	.00000012
.360	.00000043	.00000034	.00000016
.380	.00000039	.00000029	.00000015
.400	.00000032	.00000020	.00000009
.420	.00000019	.00000010	.00000005
.440	.00000010	.00000003	.00000002
.460	.00000005	-.00000001	.00000000
.480	-.00000009	-.00000008	-.00000004
.500	-.00000017	-.00000012	-.00000006

2MI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000015	.000008	.000008
.06	350.0	.000135	.000072	.000072
.08	375.0	.000202	.000108	.000108
.10	375.0	.000270	.000176	.000124
.12	375.0	.000330	.000212	.000132
.14	375.0	.000345	.000248	.000136
.16	375.0	.000360	.000268	.000140
.18	375.0	.000367	.000272	.000144
.20	375.0	.000375	.000276	.000144
.22	0.0	.000270	.000176	.000108
.24	0.0	.000173	.000108	.000072
.26	0.0	.000135	.000072	.000032
.28	0.0	.000060	.000032	.000024
.30	0.0	.000045	.000024	.000016
.32	0.0	.000030	.000016	.000008
.34	0.0	.000015	.000008	.000004
.36	0.0	.000015	.000008	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2MI3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000090	.000080	.000040
.06	700.0	.000675	.000432	.000200
.08	750.0	.000750	.000472	.000228
.10	750.0	.000780	.000480	.000232
.12	750.0	.000810	.000488	.000236
.14	750.0	.000817	.000496	.000240
.16	750.0	.000825	.000496	.000244
.18	750.0	.000832	.000504	.000248
.20	750.0	.000840	.000512	.000252
.22	0.0	.000600	.000320	.000160
.24	0.0	.000300	.000208	.000080
.26	0.0	.000150	.000160	.000040
.28	0.0	.000075	.000080	.000032
.30	0.0	.000060	.000048	.000024
.32	0.0	.000045	.000032	.000016
.34	0.0	.000030	.000024	.000008
.36	0.0	.000015	.000016	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2MI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000060	.00000053	.00000027
.020	.00000444	.00000283	.00000131
.040	.00000453	.00000283	.00000137
.060	.00000427	.00000260	.00000126
.080	.00000404	.00000239	.00000116
.100	.00000369	.00000220	.00000107
.120	.00000337	.00000198	.00000099
.140	.00000308	.00000184	.00000092
.160	.00000282	.00000170	.00000085
.180	.00000099	.00000031	.00000018
.200	-.00000067	-.00000020	-.00000025
.220	-.00000116	-.00000022	-.00000035
.240	-.00000112	-.00000047	-.00000025
.260	-.00000071	-.00000040	-.00000016
.280	-.00000036	-.00000024	-.00000009
.300	-.00000009	-.00000007	-.00000003
.320	.00000013	.00000006	.00000004
.340	.00000040	.00000018	.00000012
.360	.00000046	.00000019	.00000012
.380	.00000035	.00000015	.00000009
.400	.00000020	.00000011	.00000004
.420	.00000006	.00000006	.00000001
.440	-.00000001	.00000001	-.00000000
.460	-.00000005	-.00000002	-.00000001
.480	-.00000010	-.00000005	-.00000003
.500	-.00000013	-.00000006	-.00000003

2MI3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000090	.000080	.000040
.06	700.0	.000675	.000432	.000200
.08	750.0	.000750	.000472	.000228
.10	750.0	.000780	.000480	.000232
.12	750.0	.000810	.000488	.000236
.14	750.0	.000817	.000496	.000240
.16	750.0	.000825	.000496	.000244
.18	750.0	.000832	.000504	.000248
.20	750.0	.000840	.000512	.000252
.22	0.0	.000600	.000320	.000160
.24	0.0	.000300	.000208	.000080
.26	0.0	.000150	.000160	.000040
.28	0.0	.000075	.000080	.000032
.30	0.0	.000060	.000048	.000024
.32	0.0	.000045	.000032	.000016
.34	0.0	.000030	.000024	.000008
.36	0.0	.000015	.000016	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

?HT1

RECORD OF RAW DATA

TIME	LOAD,LBS.	D E F L E C T I O N S,IN.		
(SEC)	X=0	X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000060	.000032	.000016
.06	175.0	.000225	.000080	.000032
.08	187.5	.000240	.000160	.000056
.10	187.5	.000255	.000200	.000080
.12	187.5	.000270	.000208	.000096
.14	187.5	.000285	.000216	.000100
.16	187.5	.000300	.000224	.000104
.18	187.5	.000315	.000232	.000108
.20	187.5	.000330	.000240	.000112
.22	0.0	.000150	.000160	.000080
.24	0.0	.000120	.000080	.000040
.26	0.0	.000090	.000064	.000032
.28	0.0	.000060	.000048	.000024
.30	0.0	.000045	.000032	.000016
.32	0.0	.000030	.000024	.000012
.34	0.0	.000015	.000016	.000008
.36	0.0	.000015	.000008	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2HI1

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000160	.00000085	.00000043
.020	.00000585	.00000205	.00000081
.040	.00000569	.00000399	.00000137
.060	.00000552	.00000467	.00000188
.080	.00000537	.00000442	.00000212
.100	.00000523	.00000419	.00000202
.120	.00000511	.00000398	.00000192
.140	.00000500	.00000380	.00000184
.160	.00000490	.00000363	.00000176
.180	-.00000023	.00000122	.00000077
.200	-.00000046	-.00000085	-.00000030
.220	-.00000065	-.00000081	-.00000035
.240	-.00000083	-.00000068	-.00000034
.260	-.00000061	-.00000060	-.00000031
.280	-.00000043	-.00000033	-.00000018
.300	-.00000027	-.00000011	-.00000008
.320	.00000026	.00000007	.00000001
.340	.00000072	.00000042	.00000018
.360	.00000063	.00000051	.00000024
.380	.00000052	.00000037	.00000019
.400	.00000040	.00000025	.00000014
.420	.00000028	.00000016	.00000009
.440	.00000019	.00000008	.00000005
.460	.00000013	.00000004	.00000002
.480	-.00000011	-.00000008	-.00000004
.500	-.00000028	-.00000018	-.00000009

2HI1

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	100.0	.000060	.000032	.000016
.06	175.0	.000225	.000080	.000032
.08	187.5	.000240	.000160	.000056
.10	187.5	.000255	.000200	.000080
.12	187.5	.000270	.000208	.000096
.14	187.5	.000285	.000216	.000100
.16	187.5	.000300	.000224	.000104
.18	187.5	.000315	.000232	.000108
.20	187.5	.000330	.000240	.000112
.22	0.0	.000150	.000160	.000080
.24	0.0	.000120	.000080	.000040
.26	0.0	.000090	.000064	.000032
.28	0.0	.000060	.000048	.000024
.30	0.0	.000045	.000032	.000016
.32	0.0	.000030	.000024	.000012
.34	0.0	.000015	.000016	.000008
.36	0.0	.000015	.000008	.000004
.38	0.0	.000015	.000008	.000004
.40	0.0	.000015	.000008	.000004
.42	0.0	.000015	.000008	.000004
.44	0.0	.000015	.000008	.000004
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2HI2

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000120	.000064	.000040
.06	350.0	.000480	.000144	.000136
.08	375.0	.000600	.000272	.000168
.10	375.0	.000615	.000352	.000192
.12	375.0	.000630	.000432	.000208
.14	375.0	.000645	.000448	.000212
.16	375.0	.000660	.000464	.000216
.18	375.0	.000675	.000472	.000220
.20	375.0	.000690	.000480	.000224
.22	0.0	.000330	.000352	.000176
.24	0.0	.000210	.000272	.000136
.26	0.0	.000180	.000192	.000096
.28	0.0	.000150	.000112	.000064
.30	0.0	.000120	.000080	.000048
.32	0.0	.000090	.000064	.000032
.34	0.0	.000075	.000048	.000028
.36	0.0	.000060	.000040	.000024
.38	0.0	.000045	.000032	.000020
.40	0.0	.000030	.000024	.000016
.42	0.0	.000015	.000016	.000012
.44	0.0	.000015	.000008	.000008
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

PHI2

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000160	.00000085	.00000053
.020	.00000625	.00000184	.00000176
.040	.00000726	.00000337	.00000202
.060	.00000674	.00000411	.00000214
.080	.00000626	.00000477	.00000214
.100	.00000583	.00000451	.00000198
.120	.00000545	.00000427	.00000183
.140	.00000510	.00000395	.00000170
.160	.00000479	.00000366	.00000159
.180	-.00000033	.00000167	.00000084
.200	-.00000131	.00000061	.00000039
.220	-.00000086	-.00000019	.00000002
.240	-.00000050	-.00000083	-.00000020
.260	-.00000022	-.00000070	-.00000018
.280	-.00000001	-.00000039	-.00000017
.300	.00000033	-.00000014	-.00000003
.320	.00000061	.00000016	.00000009
.340	.00000083	.00000041	.00000019
.360	.00000052	.00000043	.00000020
.380	.00000013	.00000034	.00000017
.400	.00000003	.00000018	.00000010
.420	-.00000002	.00000008	.00000002
.440	-.00000004	-.00000000	-.00000000
.460	-.00000004	-.00000004	-.00000002
.480	-.00000010	-.00000010	-.00000005
.500	-.00000013	-.00000013	-.00000006

2HI2

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	D E F L E C T I O N S,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	200.0	.000120	.000064	.000040
.06	350.0	.000480	.000144	.000136
.08	375.0	.000600	.000272	.000168
.10	375.0	.000615	.000352	.000192
.12	375.0	.000630	.000432	.000208
.14	375.0	.000645	.000448	.000212
.16	375.0	.000660	.000464	.000216
.18	375.0	.000675	.000472	.000220
.20	375.0	.000690	.000480	.000224
.22	0.0	.000330	.000352	.000176
.24	0.0	.000210	.000272	.000136
.26	0.0	.000180	.000192	.000096
.28	0.0	.000150	.000112	.000064
.30	0.0	.000120	.000080	.000048
.32	0.0	.000090	.000064	.000032
.34	0.0	.000075	.000048	.000028
.36	0.0	.000060	.000040	.000024
.38	0.0	.000045	.000032	.000020
.40	0.0	.000030	.000024	.000016
.42	0.0	.000015	.000016	.000012
.44	0.0	.000015	.000008	.000008
.46	0.0	.000015	.000008	.000004
.48	0.0	.000015	.000008	.000004
.50	0.0	.000015	.000008	.000004

2HI3

RECORD OF RAW DATA

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000375	.000224	.000064
.06	700.0	.000825	.000736	.000240
.08	750.0	.000863	.000848	.000352
.10	750.0	.000975	.000912	.000400
.12	750.0	.001050	.000928	.000416
.14	750.0	.001088	.000944	.000424
.16	750.0	.001125	.000960	.000432
.18	750.0	.001163	.000976	.000440
.20	750.0	.001200	.000992	.000448
.22	0.0	.000525	.000640	.000320
.24	0.0	.000450	.000448	.000240
.26	0.0	.000375	.000256	.000160
.28	0.0	.000300	.000160	.000096
.30	0.0	.000225	.000128	.000080
.32	0.0	.000188	.000096	.000064
.34	0.0	.000150	.000064	.000048
.36	0.0	.000113	.000048	.000032
.38	0.0	.000075	.000032	.000024
.40	0.0	.000075	.000016	.000016
.42	0.0	.000075	.000016	.000008
.44	0.0	.000075	.000016	.000008
.46	0.0	.000075	.000016	.000008
.48	0.0	.000075	.000016	.000008
.50	0.0	.000075	.000016	.000008

2HI3

RESPONSE FUNCTIONS FROM IMPLICIT CONVOLUTION

TIME (SEC)	RESPONSE FUNCTIONS X=3.25	X=5.75	X=8.25
0.000	.00000250	.00000149	.00000043
.020	.00000527	.00000477	.00000156
.040	.00000501	.00000506	.00000216
.060	.00000526	.00000498	.00000227
.080	.00000523	.00000459	.00000215
.100	.00000496	.00000423	.00000199
.120	.00000471	.00000392	.00000184
.140	.00000449	.00000363	.00000171
.160	.00000429	.00000337	.00000159
.180	-.00000039	.00000083	.00000062
.200	-.00000036	-.00000007	.00000017
.220	-.00000032	-.00000084	-.00000016
.240	-.00000026	-.00000091	-.00000035
.260	-.00000021	-.00000057	-.00000021
.280	.00000005	-.00000030	-.00000010
.300	.00000027	-.00000009	-.00000001
.320	.00000045	.00000018	.00000006
.340	.00000058	.00000039	.00000016
.360	.00000048	.00000033	.00000015
.380	.00000040	.00000029	.00000010
.400	.00000033	.00000018	.00000007
.420	.00000027	.00000007	.00000003
.440	.00000022	.00000000	.00000001
.460	.00000020	-.00000003	-.00000000
.480	-.00000004	-.00000009	-.00000003
.500	-.00000024	-.00000011	-.00000005

2HI3

DEFLECTIONS FROM RESPONSE FUNCTIONS AND LOAD

TIME (SEC)	LOAD,LBS. X=0	DEFLECTIONS,IN. X=3.25	X=5.75	X=8.25
0.00	0.0	0.000000	0.000000	0.000000
.02	0.0	0.000000	0.000000	0.000000
.04	400.0	.000375	.000224	.000064
.06	700.0	.000825	.000736	.000240
.08	750.0	.000863	.000848	.000352
.10	750.0	.000975	.000912	.000400
.12	750.0	.001050	.000928	.000416
.14	750.0	.001088	.000944	.000424
.16	750.0	.001125	.000960	.000432
.18	750.0	.001163	.000976	.000440
.20	750.0	.001200	.000992	.000448
.22	0.0	.000525	.000640	.000320
.24	0.0	.000450	.000448	.000240
.26	0.0	.000375	.000256	.000160
.28	0.0	.000300	.000160	.000096
.30	0.0	.000225	.000128	.000080
.32	0.0	.000187	.000096	.000064
.34	0.0	.000150	.000064	.000048
.36	0.0	.000112	.000048	.000032
.38	0.0	.000075	.000032	.000024
.40	0.0	.000075	.000016	.000016
.42	0.0	.000075	.000016	.000008
.44	0.0	.000075	.000016	.000008
.46	0.0	.000075	.000016	.000008
.48	0.0	.000075	.000016	.000008
.50	0.0	.000075	.000016	.000008

APPENDIX B

STATIC LOAD PROGRAM

```
      PROGRAM STALOD(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     ***************************************************
C
C     COMPUTER PROGRAM TO PREDICT STATIC LOAD RESPONSE
C
C     ***************************************************
C
C     THE PROGRAM COMPUTES THE STEADY STATE RESPONSE. THE
C     INPUTS ARE THE LOAD MAGNITUDE AND THE PARAMETERS
C     DETERMINED FROM IMPULSE TESTS.
C     K= CONVERSION FACTOR. K= 10 IN THIS STUDY.
C     TN= TIME AT WHICH IMPULSE INPUT FIRST BECOMES NONZERO
C     TN= .04 IN THIS STUDY.
C     NS= NUMBER OF CASES ANALYZED
C     FO= FORCE MAGNITUDE IN POUNDS
C     STPRSP=STEP LOAD RESPONSE TO BE CALCULATED
C     ALPHA, BETA, AND GAMMA ARE PAVEMENT PARAMETERS
C     THE OTHER VARIABLES ARE SELFDESCRIPTIVE AND REFER TO
C     THE STATIC LOAD EQUATION IN THE TEXT
C
C
      DIMENSION LABLE(4)
      INTEGER LABLE
      REAL K
      READ(5,1) NS,K,TN
      DO 99 I=1,NS
      READ(5,2) (LABLE(J),J=1,3),FO
      READ(5,3) ALPHA,BETA,GAMMA
      DEN=BETA**2+GAMMA**2
      ROTDEN=SQRT(DEN)
      TANPHI=GAMMA/BETA
      PHI=ATAN(TANPHI)
      ARGSIN=GAMMA*TN+PHI
      STPRSP=FO*ALPHA*(K*GAMMA/DEN-(K-1.)/ROTDEN*EXP(-BETA*
     1TN)*SIN(ARGSIN))
C     MULTIPLY RESULT BY 10000 FOR PRESENTATION
      STPRSP=STPRSP*10000.
C     THE FOLLOWING IF STATEMENTS ALLOW  PRINTING CERTAIN
C     GROUPS OF RESULTS CN ONE PAGE
      IF(I.EQ. 1) GO TO 20
      IF(I.EQ. 7) GO TO 20
      IF(I.EQ.16) GO TO 20
      IF(I.EQ.25) GO TO 20
      IF(I.EQ.34) GO TO 20
      IF(I.EQ.43) GO TO 20
```

```
      WRITE(6,5) (LABLE(M),M=1,3),FO,ALPHA,BETA,GAMMA,STPRSP
      GO TO 99
   20 WRITE(6,4)
      WRITE(6,5) (LABLE(M),M=1,3),FO,ALPHA,BETA,GAMMA,STPRSP
   99 CONTINUE
C     ****************************************************
C
C     FORMAT STATEMENTS
C
C     ****************************************************
C
    1 FORMAT(I3,2F10.5)
    2 FORMAT(3A6,2X,F10.2)
    3 FORMAT(E18.8,2F18.8)
    4 FORMAT(1H1,///9X,'SERIES',8X,'LOAD',11X,'P A R A M E '
     1,'T E R S',7X,'STEP RESP'//23X,'LBS.',7X,'ALPHA',7X,
     2'BETA',6X,'GAMMA',2X,'IN.*1/10000')
    5 FORMAT(//3A6,F10.2,E13.3,3F10.3)
C
      STOP
      END
```

APPENDIX C

REPEATED LOAD PROGRAM

```
      PROGRAM REPLOD (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     ******************************************************
C
C     COMPUTER PROGRAM TO PREDICT REPEATED LOAD RESPONSE
C
C     ******************************************************
C
C     THE PROGRAM COMPUTES THE TIME-DEPENDENT RESPONSE OF
C     THE SYSTEM. THE INPUTS ARE THE RATE (OR DURATION) AND
C     THE AMPLITUDE OF THE LOADING FUNCTION, AND THE PARAMETERS
C     DETERMINED FROM IMPULSE TESTS.
C
C     K= CONVERSION FACTOR. K= 10 IN THIS STUDY.
C     TN= TIME AT WHICH IMPULSE INPUT FIRST BECOMES NONZERO.
C     TN= .04 IN THIS STUDY.
C     FO= AMPLITUDE OF REPEATED LOADING FUNCTION.
C     P= PERIOD OF LOADING.
C     ALPHA, BETA AND GAMMA ARE PAVEMENT PARAMETERS.
C     NS= NUMBER OF CASES ANALYZED.
C     NP= NUMBER OF PREDICTIONS DESIRED FOR EACH CASE.
C     N= NUMBER OF LOAD APPLICATIONS.
C     TPEAK= TIME WHEN THE LOAD IS AT PEAK POSITION AFTER N
C     LOAD APPLICATIONS.
C     RPEAK= RESPONSE WHEN THE LOAD IS AT PEAK POSITION
C     AFTER N LOAD APPLICATIONS.
C     TOFPK= TIME WHEN LOAD IS AT OFF PEAK POSITION AFTER N
C     LOAD APPLICATIONS.
C     ROFPK= RESPONSE WHEN LOAD IS AT OFF PEAK POSITION
C     AFTER N LOAD APPLICATIONS.
C     THE OTHER VARIABLES ARE SELFDESCRIPTIVE AND REFER TO
C     THE REPEATED LOAD EQUATIONS IN THE TEXT.
C
C
      DIMENSION LABLE(11)
      INTEGER LABLE
      REAL K
      READ(5,1) NS,NP,P,K,TN
      DO 200 M=1,NS
      WRITE (6,2)
      READ (5,3) (LABLE(J),J=1,11),FO
      READ(5,4) ALPHA, BETA, GAMMA
      WRITE(6,5) (LABLE(J),J=1,11)
      WRITE(6,6) FO
      WRITE(6,7) ALPHA,BETA,GAMMA
```

```
      WRITE(6,8) P
      WRITE(6,9)
      DEN=BETA**2+GAMMA**2
      RTDEN=SQRT(DEN)
      TNPHI=GAMMA/BETA
      PHI=ATAN(TNPHI)
      TWOPI=2.*3.14159265
      OMEGA=TWOPI/P
      DEN1=BETA**2+(GAMMA-OMEGA)**2
      RTDEN1=SQRT(DEN1)
      TNPHI1=(GAMMA-OMEGA)/BETA
      PHI1=ATAN(TNPHI1)
      DEN2=BETA**2+(GAMMA+OMEGA)**2
      RTDEN2=SQRT(DEN2)
      TNPHI2=(GAMMA+OMEGA)/BETA
      PHI2=ATAN(TNPHI2)
      N=1
      DO 50 I=1,NP
      TPEAK=(2*N-1)*P/2.
      TOFPK=N*P
      RPEAK= FO*ALPHA/2.*(K*GAMMA/DEN+K/2.*((GAMMA-OMEGA)/DEN1
     1+(GAMMA+OMEGA)/DEN2)-(K-1.)/RTDEN*EXP(-BETA*TN)*SIN(
     2GAMMA*TN+PHI)-(K-1.)/2.*(EXP(-BETA*TN)/TRDEN1*SIN((
     3GAMMA-OMEGA)*TN+PHI1)+EXP(-BETA*TN)/RTDEN2*SIN((GAMMA+
     4OMEGA)*TN+PHI2))-EXP(-BETA*TPEAK)/RTDEN*SIN(GAMMA*TPEAK
     5+PHI)-.5*(EXP(-BETA*TPEAK)/RTDEN1*SIN((GAMMA-OMEGA)*
     6TPEAK+PHI1)+EXP(-BETA*TPEAK)/RTDEN2*SIN((GAMMA+OMEGA)*
     7TPEAK+PHI2)))
      ROFPK= FO*ALPHA/2.*(K*GAMMA/DEN-K/2.*((GAMMA-OMEGA)/DEN1
     1+(GAMMA+OMEGA)/DEN2)-(K-1.)/RTDEN*EXP(-BETA*TN)*SIN(
     2GAMMA*TN+PHI)+(K-1.)/2.*EXP(-BETA*TN)/RTDEN1*SIN((
     3GAMMA-OMEGA)*TN+PHI1)+EXP(-BETA*TN)/RTDEN2*SIN((GAMMA+
     4OMEGA)*TN+PHI2))-EXP(-BETA*TOFPK)/RTDEN*SIN(GAMMA*TOFPK
     5+PHI)+.5*(EXP(-BETA*TOFPK)/RTDEN1*SIN((GAMMA-OMEGA)*
     6TOFPK+PHI1)+EXP(-BETA*TOFPK)/RTDEN2*SIN((GAMMA+OMEGA)*
     7TOFPK+PHI2)))
      WRITE(6,10) TPEAK,RPEAK,TOFPK,ROFPK
   50 N=N+1
  200 CONTINUE
C
C     ****************************************************
C
C     FORMAT STATEMENTS
C
C     ****************************************************
C
    1 FORMAT(2I3,3F10.5)
    2 FORMAT(1H1)
    3 FORMAT(11A6,F10.1)
    4 FORMAT(E18.8,2F18.8)
    5 FORMAT(11A6)
```

```
    6 FORMAT(///6X,'THE LOAD MAGNITUDE =',F10.1,'  LBS.')
    7 FORMAT(///6X,'THE PARAMETERS ARE',//8X,'ALPHA=',E12.3,
     1'   BETA=',F8.3,'   GAMMA=',F8.3)
    8 FORMAT(//6X,'THE PERIOD =',F10.5,'  SEC.')
    9 FORMAT(///8X,'TPEAK',4X,'PEAK RESPONSE',5X,'TOFPK',5X,
     1'OFFPK RESPONSE')
   10 FORMAT(//5X,E9.2,E14.3,E14.2,E15.3)
C
C     ******************************************************
C
      STOP
      END
```

VITA

VITA

Galal Abdalla Ali was born on January 1, 1941 in Saysab, Northern Province, Sudan. He attended the Abri Elementary and Intermediate Schools. His Secondary School education was completed at Wadi Seidna, Omdurman in April 1960. He received his Bachelor of Science degree in Engineering from University of Khartoum, Khartoum, with First Class Honors in April 1966.

After graduation, the author was employed as a Demonstrator in the Department of Civil Engineering, University of Khartoum.

In September 1966, he came to the United States of America and received his degree of Master of Science in Highway Engineering from Northwestern University in August, 1967.

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Mr. Ali is an associate member of the American Society of Civil Engineers, a student member of the Institute of Traffic Engineers and an individual supporting member of the Highway Research Board.

He has to his credit one technical paper.

Mr. Ali is a citizen of Sudan. He is married and has two children.